Cooperative Tracker for LTI MAS

MAS $\dot{x}_i = Ax_i + Bu_i$

control $u_i = cK \sum_{j \in N_i} a_{ij}(x_j - x_i) + g_i(x_0 - x_i)$

Leader dynamics $\dot{x}_0 = Ax_0 + Bu_0$

closed-loop local systems

$$\dot{x}_i = Ax_i + cBK\left[\sum_j a_{ij}(x_j - x_i) + g_i(x_0 - x_i)\right]$$

1/ global closed-loop dynamics

$$\dot{x} = \begin{bmatrix} \dot{x}_1 \\ \vdots \\ \dot{x}_i \\ \vdots \\ \dot{x}_N \end{bmatrix} = \begin{bmatrix} A & & \\ & \ddots & \\ & & A \end{bmatrix} x - c \begin{bmatrix} (d_1 + g_1)BK & & \\ & \ddots & \\ & & BK(d_N + g_N) \end{bmatrix}$$

$$+ c \begin{bmatrix} a_{ij} BK \end{bmatrix} x + c \begin{bmatrix} g_1 BK & & \\ & \ddots & \\ & & g_N BK \end{bmatrix} \begin{bmatrix} x_0 \\ \vdots \\ x_0 \end{bmatrix}$$

$$\dot{x} = \left(I_N \otimes A - c(L+G) \otimes BK\right) x + c(G \otimes BK)\, \underline{x_0}$$

add L: $(L \otimes I_m)\, \underline{x_0} = c\,(\underline{1}_N \otimes x_0)$

$$\dot{x} = \mathcal{A}_c\, x + c((L+G) \otimes BK)\, \underline{x_0}$$

2- Proof of synchronization

Find error dynamics.

Def $\delta_i = x_i - x_0$

Global —

$$\dot{\delta} = \begin{bmatrix} \dot{\delta}_1 \\ \vdots \\ \dot{\delta}_N \end{bmatrix} = \dot{x} - \dot{\underline{x}}_0 = A_c x + c((L+G)\otimes BK)\underline{x}_0 - (I_N \otimes A)\underline{x}_0 - (I_N \otimes B)\underline{u}_0$$

$((I_N \otimes A) - c(L+G)\otimes BK)$

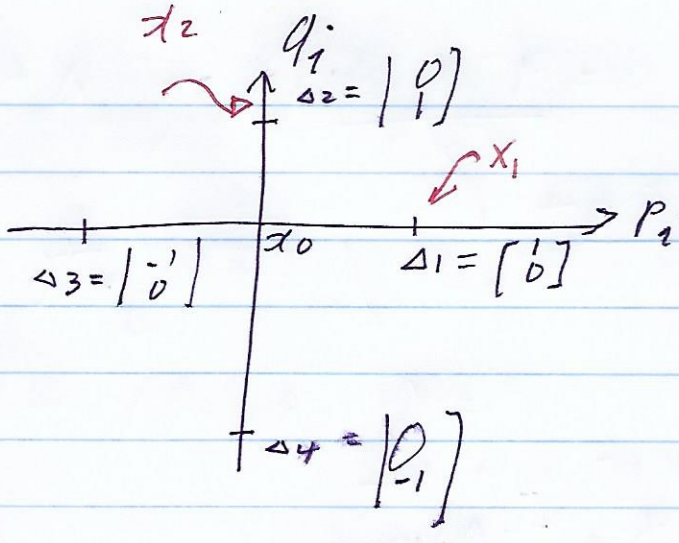
$$\begin{bmatrix} u_0 \\ u_0 \\ \vdots \\ u_0 \end{bmatrix} = \underline{1}_N \otimes u_0$$

$$\dot{\delta} = \left((I_N \otimes A) - c(L+G)\otimes BK\right)\delta + (I_N \otimes B)\underline{u}_0 \qquad (*)$$

Thm Let A_c be stable and u_0 be bounded. Then $(*)$ is a stable system with bounded disturbance input, & bounded synchronization errors result. (UUB).

Moreover if $u_0 = 0$, then we get exact synchronization.

FORMATION CONTROL



$$= \begin{bmatrix} p_{x_1} \\ q_{y_1} \end{bmatrix}$$

$$\begin{bmatrix} p_i \\ q_i \end{bmatrix}$$

$x_1 \rightarrow (x_0 + \Delta_1)$ $\qquad x_0 - (x_1 - \Delta_1)$

$x_i \rightarrow x_0 + \Delta_i$ $\qquad x_0 - (x_i - \Delta_i) - x_0 - (x_j - \Delta_j) = -x_j + (x_0 + \Delta_j)$

$$\epsilon_i = \sum a_{ij} \left((x_j - \Delta_j) - (x_i - \Delta_i) \right) + g_i \left(x_0 - (x_i - \Delta_i) \right)$$

$-x_0$ $\qquad +x_0$

$$\tilde{x}_i = (x_i - \Delta_i)$$

$$\sum a_{ij} (\tilde{x}_j - \tilde{x}_i) + g_i (x_0 - \tilde{x}_i)$$

$$x_i \rightarrow x_0 + \Delta_i$$

$$x_i = \begin{bmatrix} p_i \\ q_i \end{bmatrix} \qquad \dot{x}_i = v_i = \begin{bmatrix} \dot{p} \text{ vel} \\ \dot{q} \text{ vel} \end{bmatrix}$$

$$\dot{v}_i = u_i = \begin{bmatrix} \dot{p} \text{ accel} \\ \dot{q} \text{ accel} \end{bmatrix}$$

$$\dot{z}_i = \begin{bmatrix} \dot{x}_i \\ \dot{v}_i \end{bmatrix} = \begin{bmatrix} 0 & I_2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_i \\ v_i \end{bmatrix} + \begin{bmatrix} 0 \\ I_2 \end{bmatrix} u_i = A z_i + B u_i$$

$$z_i = \begin{bmatrix} x_i \\ v_i \end{bmatrix}$$

3- Formation Control Application

MAS $\dot{x}_i = v_i$, $\dot{v}_i = u_i$ Leader $\dot{x}_0 = v_0$, $\dot{v}_0 = u_0$

Formation control protocol

$$u_i = c\left[k_p\left(\sum a_{ij}(x_j - \Delta_j - x_i + \Delta_i) + g_i(x_0 - x_i + \Delta_i)\right)\right.$$
$$\left. + k_d\left(\sum a_{ij}(v_j - v_i) + g_i(v_0 - v_i)\right)\right]$$

Define $z_i = \begin{bmatrix} x_i \\ v_i \end{bmatrix}$

Then MAS is $\dot{z}_i = \begin{bmatrix} 0 & I \\ 0 & 0 \end{bmatrix} z_i + \begin{bmatrix} 0 \\ I \end{bmatrix} u_i \equiv A z_i + B u_i$

leader $\dot{z}_0 = \begin{bmatrix} 0 & I \\ 0 & 0 \end{bmatrix} z_0 + \begin{bmatrix} 0 \\ I \end{bmatrix} u_0 \equiv A z_0 + B u_0$

control is

$$v_i = c\begin{bmatrix} k_p & k_d \end{bmatrix}\left[\sum_{j \in N_i} a_{ij}\left(\begin{bmatrix} x_j - \Delta_j - x_i + \Delta_i \\ v_j - v_i \end{bmatrix}\right.\right.$$
$$\left.\left. + g_i \begin{bmatrix} x_0 - x_i + \Delta_i \\ v_0 - v_i \end{bmatrix}\right)$$

@/ stability of Formation

Def. errors $\tilde{x}_i = x_i - \Delta_i - x_0$,
$\tilde{V}_i = V_i - V_0$, $\Delta_i = \underline{const}$

$$\tilde{z}_i = \begin{vmatrix} \tilde{x}_i \\ \tilde{V}_i \end{vmatrix} = (z_i - z_0) - \begin{bmatrix} \Delta_i \\ 0 \end{bmatrix} \equiv (z_i - z_0) - \bar{\Delta}_i$$

$$\dot{\tilde{z}}_i = \begin{vmatrix} \dot{\tilde{x}}_i \\ \dot{\tilde{V}}_i \end{vmatrix} = \begin{vmatrix} \dot{x}_i - \dot{x}_0 \\ \dot{V}_i - \dot{V}_0 \end{vmatrix} = \dot{z}_i - \dot{z}_0 \quad , \Delta_i = const$$

$$= A z_i + B U_i - (A z_0 + B V_0)$$

$$\dot{\tilde{z}}_i = A(z_i - z_0 - \bar{\Delta}_i) + \underline{A \bar{\Delta}_i} + B(U_i - V_0)$$

$$= A \tilde{z}_i + \underline{A \bar{\Delta}_i} + B(U_i - U_0)$$

$$U_i \equiv CK \sum_j a_{ij} \begin{vmatrix} \tilde{x}_j - \tilde{x}_i \\ \tilde{V}_j - \tilde{V}_i \end{vmatrix} - g_i \begin{bmatrix} \tilde{x}_i \\ \tilde{V}_i \end{bmatrix}$$

$$\dot{\tilde{z}}_i = A \tilde{z}_i + \underline{A \bar{\Delta}_i} \qquad U_i = \left(CK \sum_j a_{ij} (\tilde{z}_j - \tilde{z}_i) - g_i \tilde{z}_i \right)$$

$$+ cBK \left(\sum_j a_{ij} (\tilde{z}_j - \tilde{z}_i) - g_i \tilde{z}_i \right) - B U_0$$

Global error dynamics

$$\dot{\tilde{z}} = \underbrace{\left((I_N \otimes A) - c\left((L+G)\otimes BK\right)\right)}_{A_c}\tilde{z} - (I_N \otimes B)\,u_0$$

$$(I_N \otimes A)\bar{\Delta}_i = 0$$

so, if A_c is stable, agents synchronize w.r.t. B to leader if $u_0 \neq 0$.

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b. Simulate Formation

$$\dot{z}_i = Az_i + Bu_i \quad , \quad \dot{z}_0 = Az_0 + Bu_0$$

$$\dot{z} = (I_N \otimes A)z + (I_N \otimes B)u .$$

$$u_i = cK \sum_j \left[a_{ij}\begin{bmatrix} x_j - x_i \\ v_j - v_i \end{bmatrix} + g_i \begin{bmatrix} x_0 - x_i \\ v_0 - v_i \end{bmatrix}\right]$$

$$+ cK\left(\sum_j a_{ij}\begin{bmatrix} -\Delta_j + \Delta_i \\ 0 \end{bmatrix} + g_i \begin{pmatrix} \Delta_i \\ 0 \end{pmatrix}\right)$$

$\uparrow \bar{\Delta}_i$

$$u = -\Big(c(L+G)\otimes K\Big)z - (G \otimes K)z_0$$

$$+ \quad c(L+G)\otimes K \begin{bmatrix} \bar{\Delta}_1 \\ \bar{\Delta}_2 \\ \vdots \end{bmatrix}$$

overall

$$\dot{\bar{z}} = \left((I_N \otimes A) - c(L+G) \otimes BK\right) \bar{z} + c(L+G) \otimes BK\bar{\Delta} + c((L+G) \otimes BK)\bar{x}_0$$

offsets $\bar{\Delta} = \begin{bmatrix} \bar{\Delta}_1 \\ \bar{\Delta}_2 \\ \vdots \\ \bar{\Delta}_N \end{bmatrix} = \begin{bmatrix} \Delta_1 \\ 0 \\ \Delta_2 \\ 0 \\ \vdots \\ \Delta_N \\ 0 \end{bmatrix}$

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Formation Control

$$\begin{cases} \dot{r}_i = v_i \\ \dot{v}_i = u_i \end{cases} \qquad \dot{r}_0 = v_0 \qquad \dot{v}_0 = u_0 \qquad r_i \in R^2 \quad r_0 \in R^2$$

$$X_i = \begin{bmatrix} r_i \\ v_i \end{bmatrix} \in R^4 \qquad n = 4$$

$$\dot{X}_i = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} X_i + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_i$$

$$= AX_i + BU_i$$

$$X = \begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_N \end{bmatrix} \qquad \dot{X} = \begin{bmatrix} A & & \\ & A & \\ & & \ddots \\ & & & A \end{bmatrix} X + \begin{bmatrix} B & & \\ & B & \\ & & \ddots \\ & & & B \end{bmatrix} U \qquad U = \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_N \end{bmatrix}$$

$$\therefore \dot{X} = (I_N \otimes A) X + (I_N \otimes B) U$$

$$u_i = c k_p \left[\sum_{j \in N_i} a_{ij} (r_j - \Delta_j - r_i + \Delta_i) + g_i (r_0 - r_i + \Delta_i) \right]$$

$$+ c k_d \left[\sum_{j \in N_i} a_{ij} (v_j - v_i) + g_i (v_0 - v_i) \right]$$

$$= c [k_p \ k_d] \left[\sum_{j \in N_i} a_{ij} \left(\begin{bmatrix} r_j \\ v_j \end{bmatrix} - \begin{bmatrix} r_i \\ v_i \end{bmatrix} \right) + g_i \left(\begin{bmatrix} r_0 \\ v_0 \end{bmatrix} - \begin{bmatrix} r_i \\ v_i \end{bmatrix} \right) \right]$$

$$- c k_p \left[\sum_{j \in N_i} a_{ij} (\Delta_j - \Delta_i) + g_i (-\Delta_i) \right]$$

$$= c K \left[\sum_{j \in N_i} a_{ij} (X_j - X_i) + g_i (X_0 - X_i) \right]$$

$$- c k_p \left[\sum_{j \in N_i} a_{ij} (\Delta_j - \Delta_i) + g_i (-\Delta_i) \right]$$

$$u = -c\left[(L+G)\otimes k\right)X - (G\otimes k)\underline{X}_0\,] \quad \text{☆}$$
$$+ c\left[(L+G)\otimes k_p\right)\Delta$$

$$\dot{X} = (I_N \otimes A)X + (I_N \otimes B)u$$

$$\dot{X} = \left[(I_N\otimes A) - c\,((L+G)\otimes BK)\right)X$$
$$+ c\,((L+G)\otimes BK)\,\underline{X}_0 + c\,((L+G)\otimes Bk_p)\,\Delta \quad \text{☆}$$

$$X = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{bmatrix} \quad u = \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_N \end{bmatrix} \quad \underline{X}_0 = \begin{bmatrix} x_0 \\ x_0 \\ \vdots \\ x_0 \end{bmatrix} \quad \Delta = \begin{bmatrix} \Delta_1 \\ \Delta_2 \\ \vdots \\ \Delta_N \end{bmatrix}$$

$$\Downarrow$$

$$X_i = \begin{bmatrix} \eta_i \\ v_i \end{bmatrix} \in R^4$$

T1 $A_c = (I_N \otimes A) - c((L+G) \otimes BK)$

is stable if

$A - c\lambda_i BK$ is stable $\forall \lambda_i$, e-val of $L+G$

T2 A_c is stable if

$K = R^{-1} B^T P$

where $A^T P + PA + Q - PBR^{-1}B^T P = 0$

some $Q, R > 0$

and $c > \frac{1}{2 \min_{Re} \lambda_i (L+G)}$

kron( )