Cooperative Tracker

$$\dot{x}_i = v_i = \sum_{j \in N_i} a_{ij}(x_j - x_i) + g_i(x_0 - x_i) = \epsilon_i$$

$$\dot{x}_i = \sum_j a_{ij} x_j - \left(\sum_j a_{ij}\right) x_i + g_i x_0 - g_i x_i$$

$$- d_i x_i$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_i \\ \vdots \end{bmatrix} = -\begin{bmatrix} d_1 & & \\ & d_2 & \\ & & d_i \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_i \end{bmatrix} + \begin{bmatrix} 0 & a_{1j} & \\ & 0 & a_{2j} \\ & & \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_j \\ \vdots \end{bmatrix}$$

$\dot{x}$ $\qquad$ x

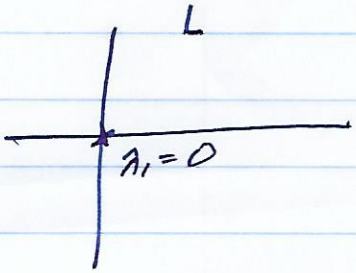
$$- \begin{bmatrix} g_1 & & \\ & g_2 & \\ & & g_i \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_i \end{bmatrix} + \begin{bmatrix} g_1 & & \\ & g_2 & \\ & & g_i \end{bmatrix} \begin{bmatrix} x_0 \\ x_0 \\ x_0 \end{bmatrix}$$

$$\underline{x}_0 = \underline{1}\, x_0$$

$$\dot{x} = -Dx + Ax - Gx + G\underline{x}_0$$

$$\dot{x} = -(D - A)x - Gx + G\underline{x}_0$$

$$\dot{x} = -(L + G)x + (L + G)\underline{x}_0$$



Global error $\qquad \delta = \begin{bmatrix} \delta_1 \\ \delta_2 \\ \delta_i \\ \delta_N \end{bmatrix} = \begin{bmatrix} x_0 - x_1 \\ x_0 - x_2 \\ x_0 - x_i \end{bmatrix} = \underline{x}_0 - x$

$$\dot{\delta} = \dot{x}_0 - \dot{x}$$

leader $\dot{x}_0 = 0$

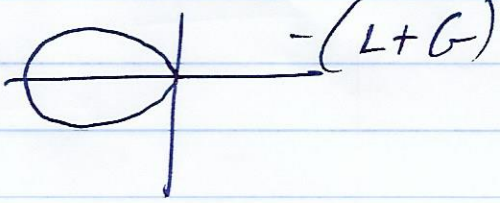
Note $L\underline{x}_0 = L\underline{1}x_0 = 0$

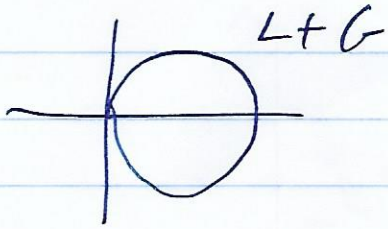
$$L\underline{x_0} = L\underline{1}x_0$$

$$\dot{\delta} = \dot{x_0} - \dot{x}$$
$$\dot{\delta} = 0 + (L+G)x - (L+G)\underline{x_0}$$
$$\dot{\delta} = -(L+G)\delta$$

need $\delta(t) = x_0(t) - x(t) \to 0$

need $-(L+G)$ stable





Thm. 1) L has 1 pole at $\lambda_1 = 0$ $\leftrightharpoons$

2) ~~Let~~ $\exists$ spanning tree

$\leftrightharpoons \lambda_2 \neq 0$

3) Let $g_i \neq 0$ for any root node i

Then $-(L+G)$ is stable

$$\epsilon_i = \sum a_{ij}(x_j - x_i)$$

local neighborhood error

$$\varepsilon_i = \sum a_{ij} (x_j - x_i) + g_i (x_0 - x_i)$$

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_N \end{bmatrix} = -\begin{bmatrix} d_1 & & & \\ & -d_2 & & \\ & & \ddots & \\ & & & \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{bmatrix} + \begin{bmatrix} 0 & a_{12} & \cdots & \\ & & & \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{bmatrix}$$

$$- \begin{bmatrix} g_1 & & & \\ & g_2 & & \\ & & \ddots & \\ & & & \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{bmatrix} + \begin{bmatrix} g_1 & & & \\ & g_2 & & \\ & & \ddots & \\ & & & \end{bmatrix} \begin{bmatrix} x_0 \\ x_0 \\ \vdots \\ x_0 \end{bmatrix}$$

$$\varepsilon = -Dx + Ax - Gx + G\underline{x_0}$$

$$= -(L+G)x + (L+G)\underline{x_0}$$

$$\varepsilon = (L+G)(\underline{x_0} - x) = (L+G)\,\delta$$

δ Global error

$$\delta_i = x_0 - x_i$$