Cooperative tracker

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$$\dot{x}_i = Ax_i + Bv_i \quad , x_i \in R^n, \; v_i \in R^m, \; i = 1, N$$

$$v_i = cK \sum_{j \in N_i} a_{ij}(x_j - x_i) + g_i(x_0 - x_i) = cK\epsilon_i$$

$$\dot{x}_i = Ax_i + cBK\left(-d_i x_i + \sum a_{ij} x_j + g_i x_0 - g_i x_i\right)$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_i \end{bmatrix} = \dot{x} = \begin{bmatrix} A & & \\ & A & \\ & & A \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_i \end{bmatrix} - c \begin{bmatrix} d_1 BK & & \\ & \ddots & \\ & & d_i BK \end{bmatrix} \begin{bmatrix} x_1 \\ x_i \end{bmatrix}$$

$$+ c \begin{bmatrix} a_{ij} BK \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_i \end{bmatrix} + c \begin{bmatrix} g_1 BK & & \\ & g_2 BK & \\ & & \ddots \end{bmatrix} \begin{bmatrix} x_0 \\ x_0 \end{bmatrix}$$

$$\mp c \begin{bmatrix} g_1 BK & & \\ & g_i BK & \\ & & \ddots \end{bmatrix} \begin{bmatrix} x_1 \\ x_i \end{bmatrix}$$

$$\dot{x} = (I_N \otimes A)\, x - c(D \otimes BK)x + c(A \otimes BK)x + c\, G \otimes BK \underbrace{x_0}_{I_N \otimes x_0} - c(G \otimes BK)x$$

$$= (I_N \otimes A)\, x - c(L \otimes BK)x - c(G \otimes BK)x + c\, G \otimes BK\, x_0$$

$$\dot{x} = (I_N \otimes A)x - c(L+G) \otimes BKx + c(\underbrace{(L+G)}_{C} \otimes BK)x_0$$

Leader Dynamics , $\dot{x}_0 = Ax_0 + Bv_0$, $\delta_i = x_i - x_0$

δ Global error $\delta = \begin{vmatrix} \delta_1 \\ \delta_i \end{vmatrix} = x - x_0$

recall $\delta = \; . \, (L+G)\epsilon$, $\epsilon \to 0 \leftrightharpoons \delta \to 0$

Error Dynamics

$$\dot{\delta} = \dot{x} - \dot{x}_0$$

$$= (I_N \otimes A)x - c[(L+G)\otimes BK](x - x_0) - (I_N \otimes A)x_0$$

$$\dot{\delta} = (I_N \otimes A)\delta - c(L+G)\otimes BK)\delta$$

$$\dot{\delta} = A_c\,\delta$$

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single agent state feedback

$$\dot{x} = Ax + Bu, \quad u = -Kx$$

closed-loop

$$\dot{x} = (A - BK)x$$

