# Discrete-Time Systems

$$x_i(k+1) = x_i(k) + v_i(k)$$

$$x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_i \\ x_N \end{bmatrix}, \quad x_{k+1} = x_k + v_k$$

1/ Perron DT consensus

$$v_i(k) = \epsilon \sum_{j} a_{ij} (x_j(k) - x_i(k))$$

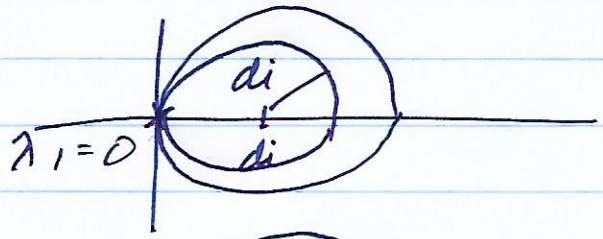
$$v_k = -\epsilon L x_k$$

$$x_{k+1} = (I - \epsilon L) x_k$$

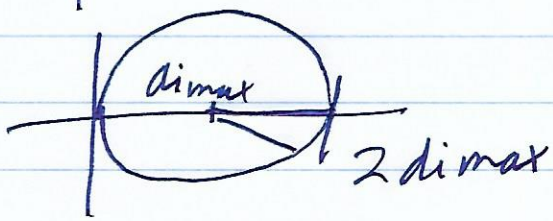
stable if poles of $I - \epsilon L$ are IUC inside unit circle

$$L = \begin{bmatrix} d_1 & & \\ & d_i & \\ & & \ddots \end{bmatrix} - \begin{bmatrix} 0 & a_{12} & \cdots & a_{1j} \\ & 0 & & \\ & & \ddots & \\ & & & 0 \end{bmatrix}$$

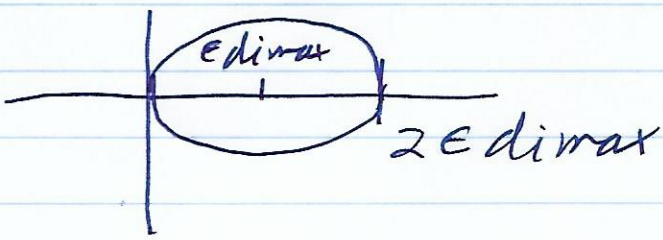
Geršgorin Circle Criterion

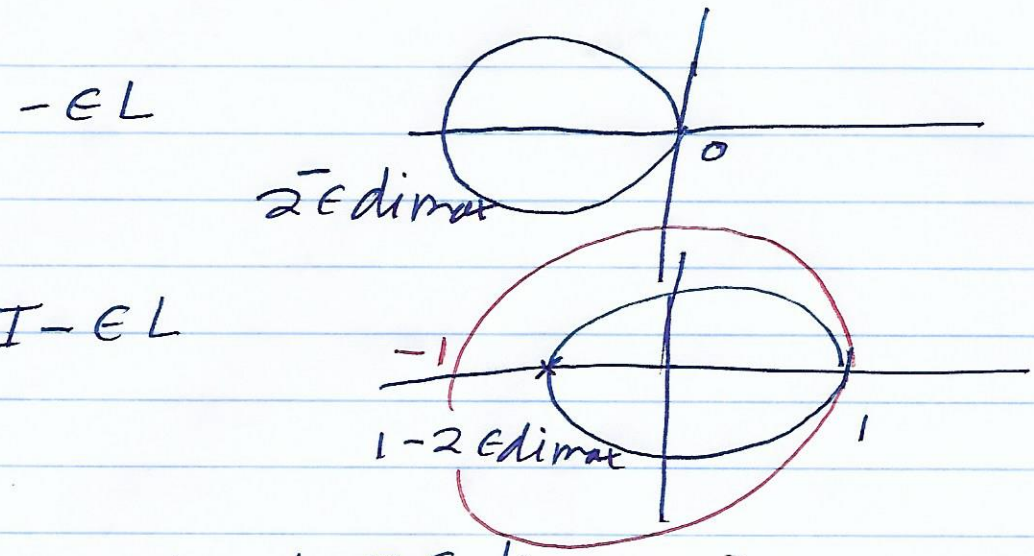



L




$\epsilon L$






$-1 < 1 - 2\epsilon d_{max}$

$2\epsilon d_{max} < 2$

$\epsilon < \frac{1}{d_{max}}$ for DT stability of Perron DT systems

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$P = I - \epsilon L$

$x_{k+1} = (1 - \epsilon L) x_k$

at steady-state

$x_{k+1} = x_k$

$x_{ss} = (1 - \epsilon L) x_{ss}$

$x_{ss} = \begin{vmatrix} 1 \\ 1 \\ 1 \end{vmatrix} c = \begin{vmatrix} x_1 \\ x_2 \\ x_i \\ \vdots \end{vmatrix}$

$\underline{1} = (1 - \epsilon L)\underline{1}$

i.e. row sum of $(1 - \epsilon L)$ must be 1

row stochastic matrix - Perron Matrix

is it? $(1 - \epsilon L)\underline{1} = \underline{1} - \epsilon L \underline{1} = \underline{1}$

c same as CT case, $c = \sum w_i^T x_i(0)$

$\dot{x} = -Lx$

at SS $\dot{x} = 0$

$\therefore -L1c = 0$

$\Rightarrow$ consensus

2/ $x_i(k+1) = x_i(k) + v_i(k)$

Normalized DT consensus

$$v_i(k) = \frac{1}{1+d_i}\left(\sum_{j\in N_i} a_{ij}\,(x_j(k) - x_i(k))\right)$$

Global $v(k) = -(I+D)^{-1}Lx$

$$(I+D)^{-1} = \begin{bmatrix} \frac{1}{1+d_1} & & \\ & \frac{1}{1+d_2} & \\ & & \ddots \end{bmatrix}$$

$$x_{k+1} = x_k + v_k = (I - (I+D)^{-1}L)\,x_k$$

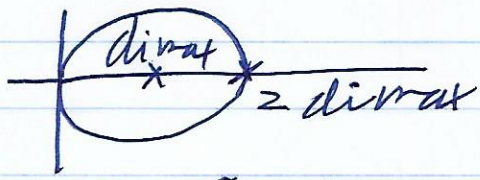
$$= Fx_k$$

$$F = I - (I+D)^{-1}L = (I+D)^{-1}(I+D-L)$$

$$= (I+D)^{-1}(I+D-(D-A)) = (I+D)^{-1}(I+A)$$

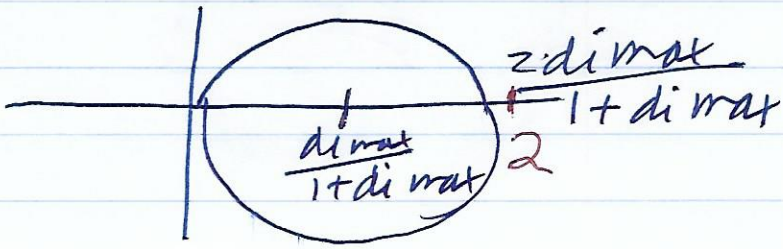
when is $x_{k+1} = (I - (I+D)^{-1}L)\,x_k$ stable?

$L$

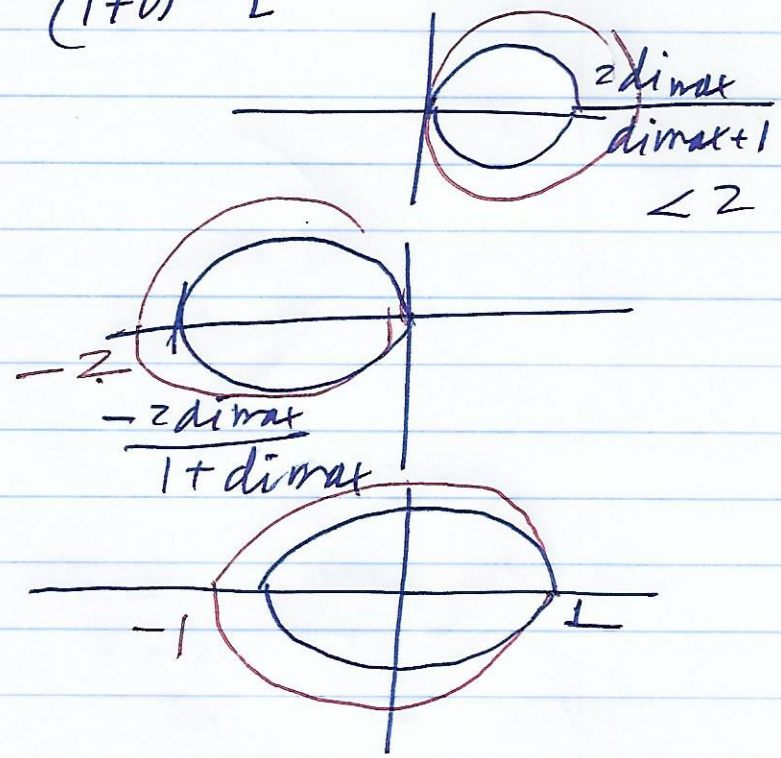



$$(I+D)^{-1}L = \begin{bmatrix} \frac{d_i}{1+d_i} & & \\ & \frac{d_i}{1+d_i} & \\ & & \ddots \end{bmatrix} - \begin{bmatrix} \frac{a_{ij}}{1+d_i} & & \\ & \frac{a_{ij}}{1+d_i} & \\ & & \ddots \end{bmatrix}$$

$$(I+D)^{-1}(D-A)$$



$(1+D)^{-1}L$




$-(1+D)^{-1}L$

$I-(1+D)^{-1}L$

Is $x_{k+1} = (1-(1+D)^{-1}L)\,x_k \equiv Fx_k$

Is $F = (1-(1+D)^{-1}L)$ row stochastic?

If so $\underline{1} = (1-(1+D)^{-1}L)\,\underline{1}$

& $x_{ss} = (1-(1+D)^{-1}L)\,x_{ss}$

$x_{k+1}$ $x_k$

where $x_{ss} = c\begin{bmatrix}1\\ \vdots \\ 1\end{bmatrix} = c\,\underline{1}$

check $(1-(1+D)^{-1}L)\,\underline{1} = \underline{1} - (1+D)^{-1}L\,\underline{1}$

$= \underline{1}$

F is row stochastic