F.L. Lewis
National Academy of Inventors

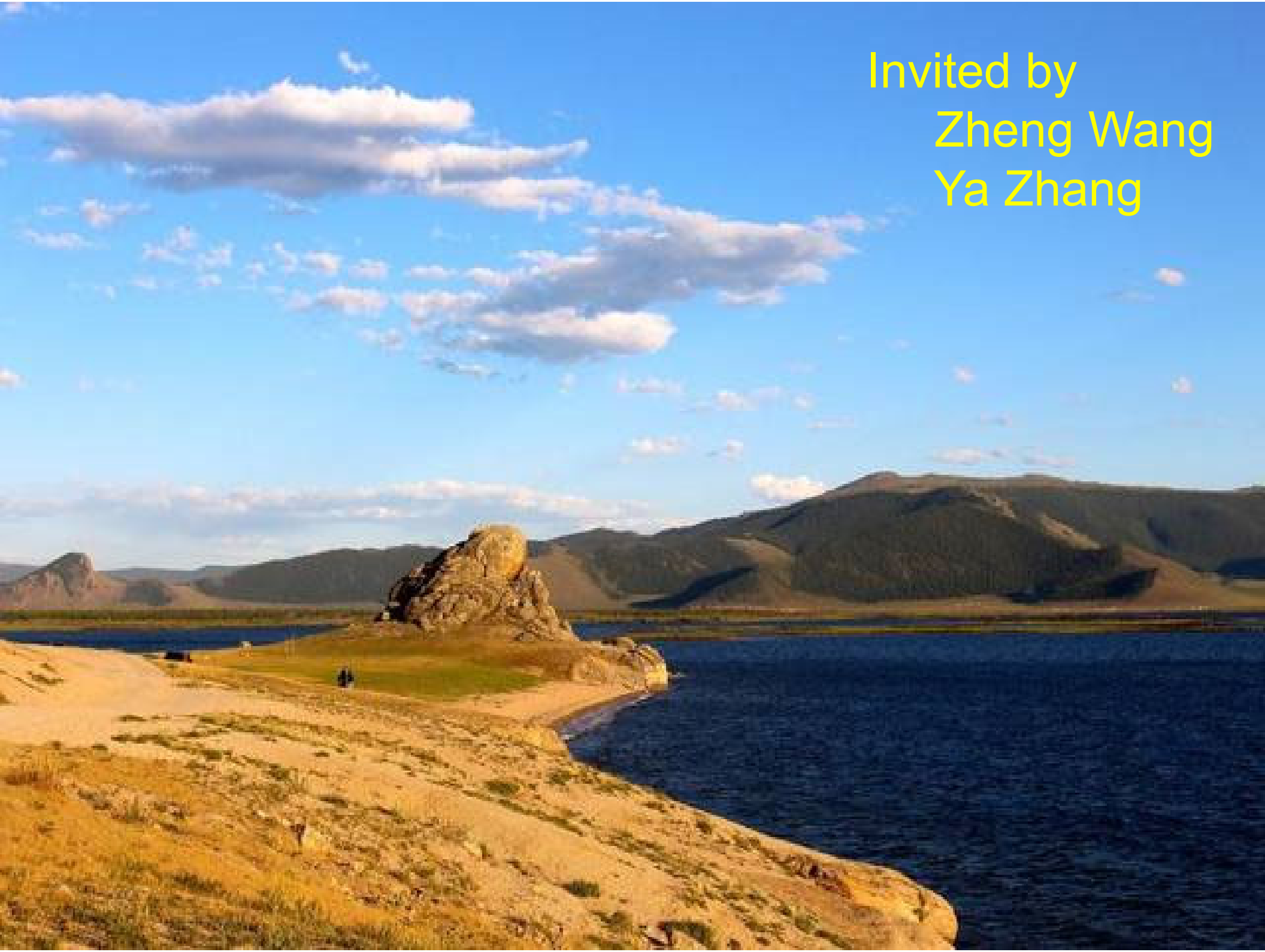
Moncrief-O'Donnell Chair, UTA Research Institute (UTARI)
The University of Texas at Arlington, USA

# Cooperative Control of Multi-Agent Systems on Communication Graphs

Supported by :
NSF
ONR
ARO/TARDEC

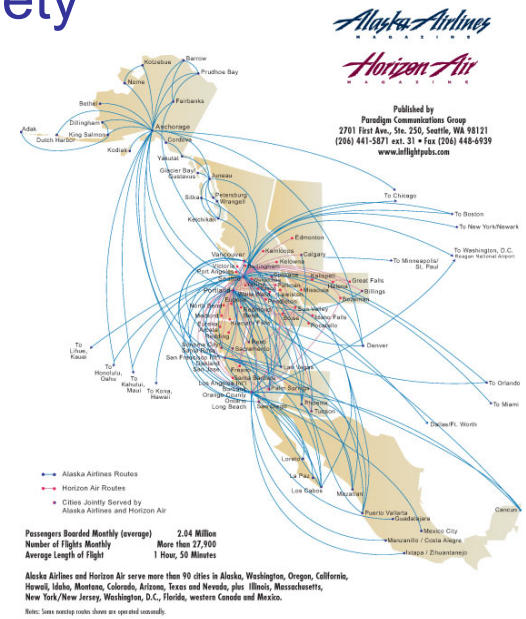


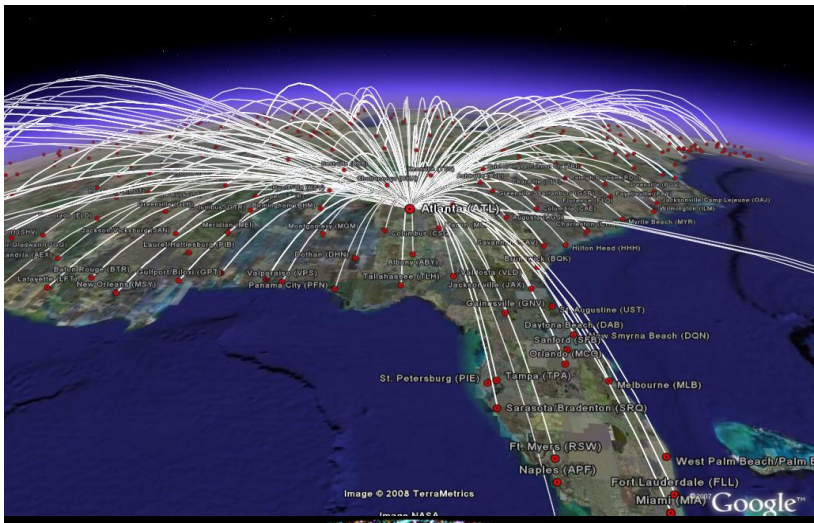

Talk available online at
https://lewisgroup.uta.edu


Invited by
Zheng Wang
Ya Zhang

# Patterns in Nature and Society



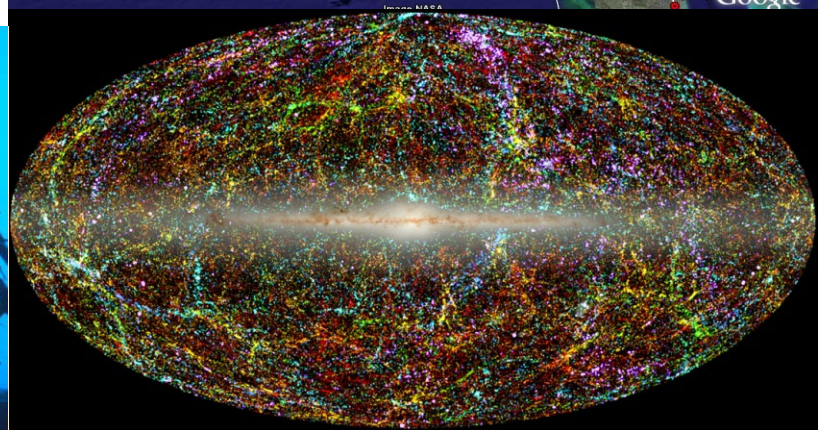



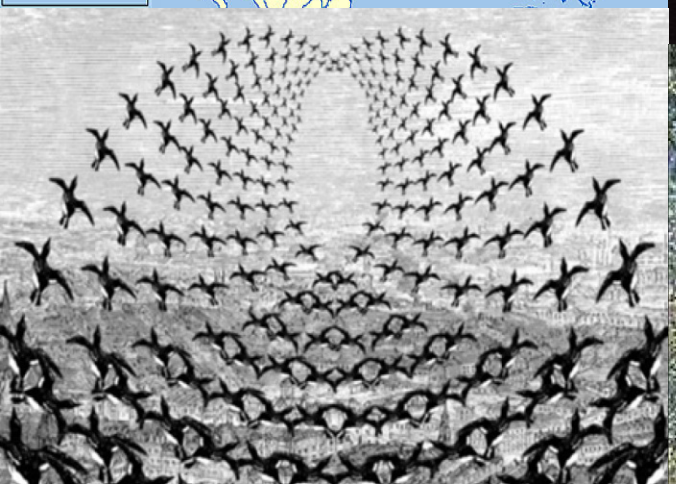





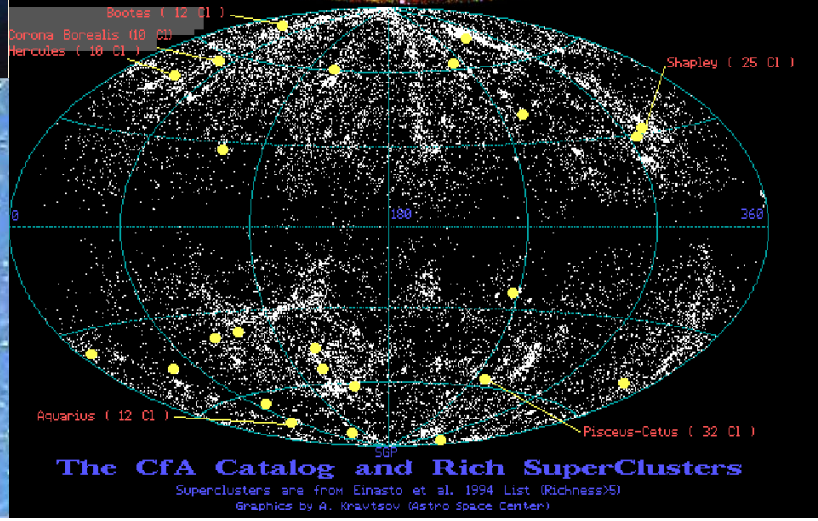




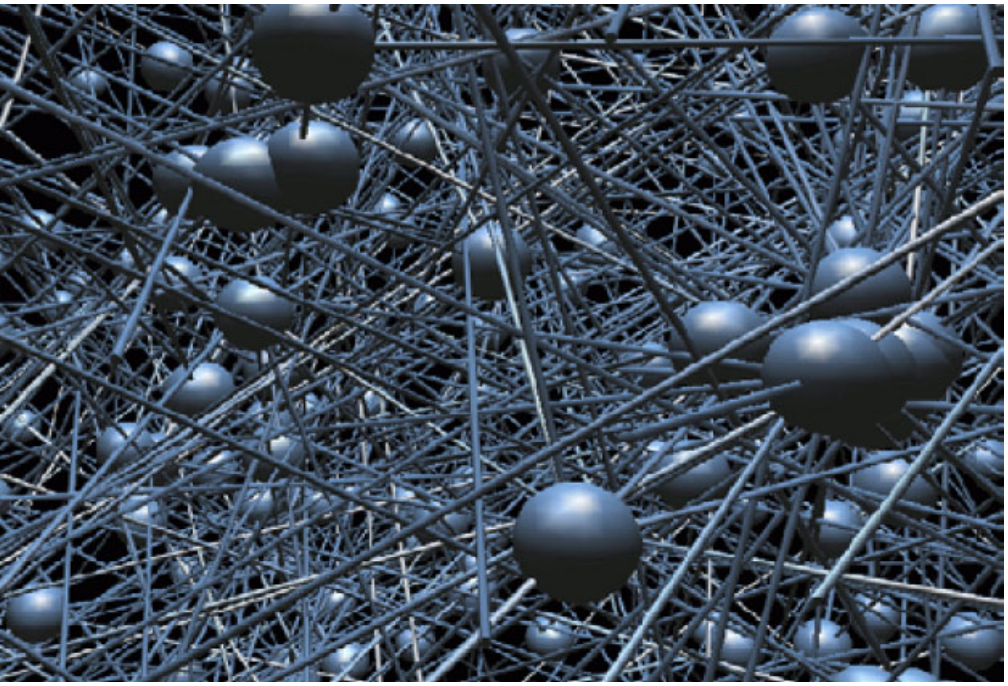


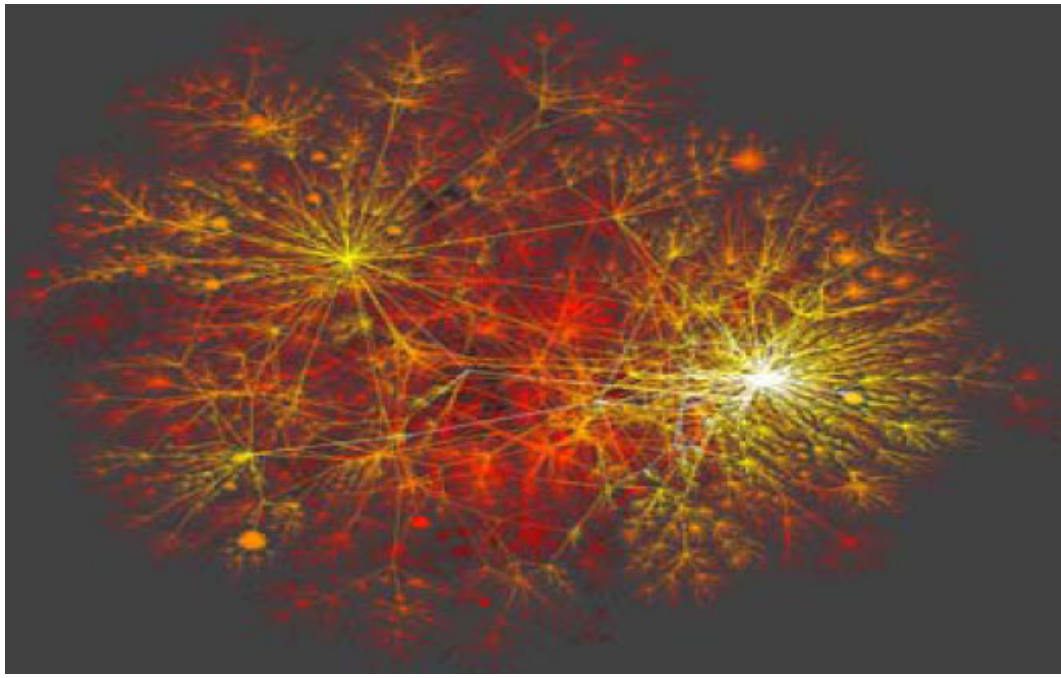


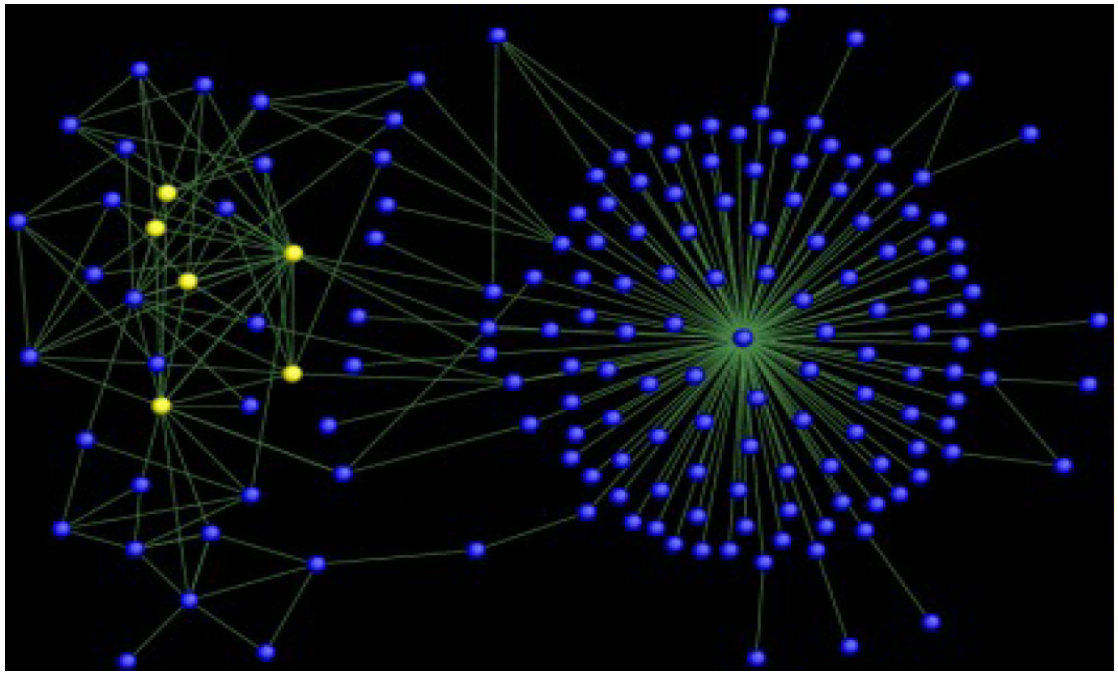




# 1. Natural and biological structures





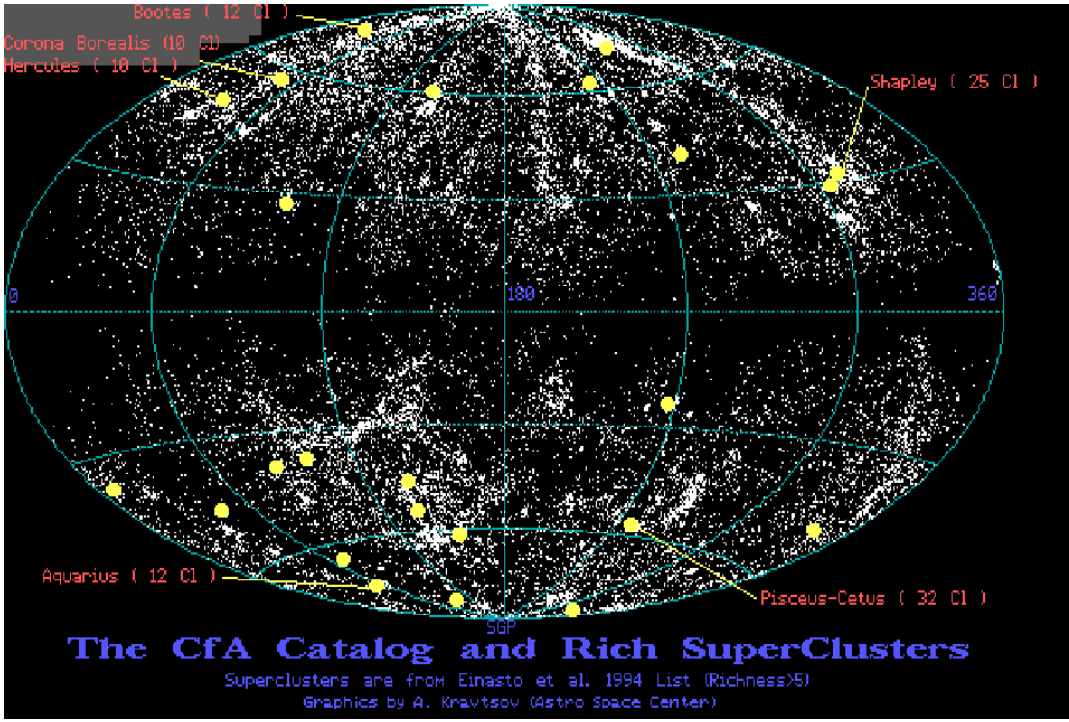


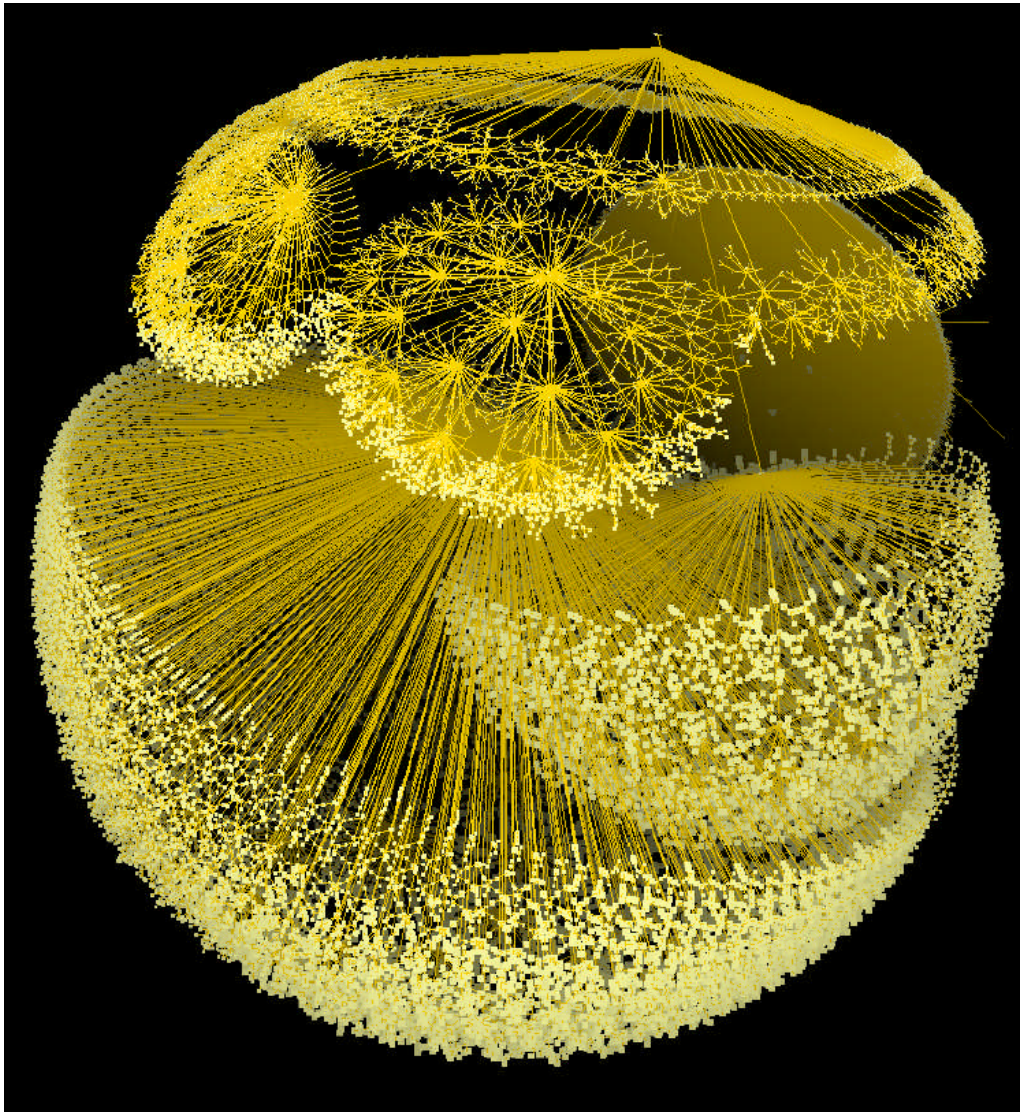
Many of the beautiful pictures are from a lecture by **Ron Chen**, City U. Hong Kong Pinning Control of Graphs

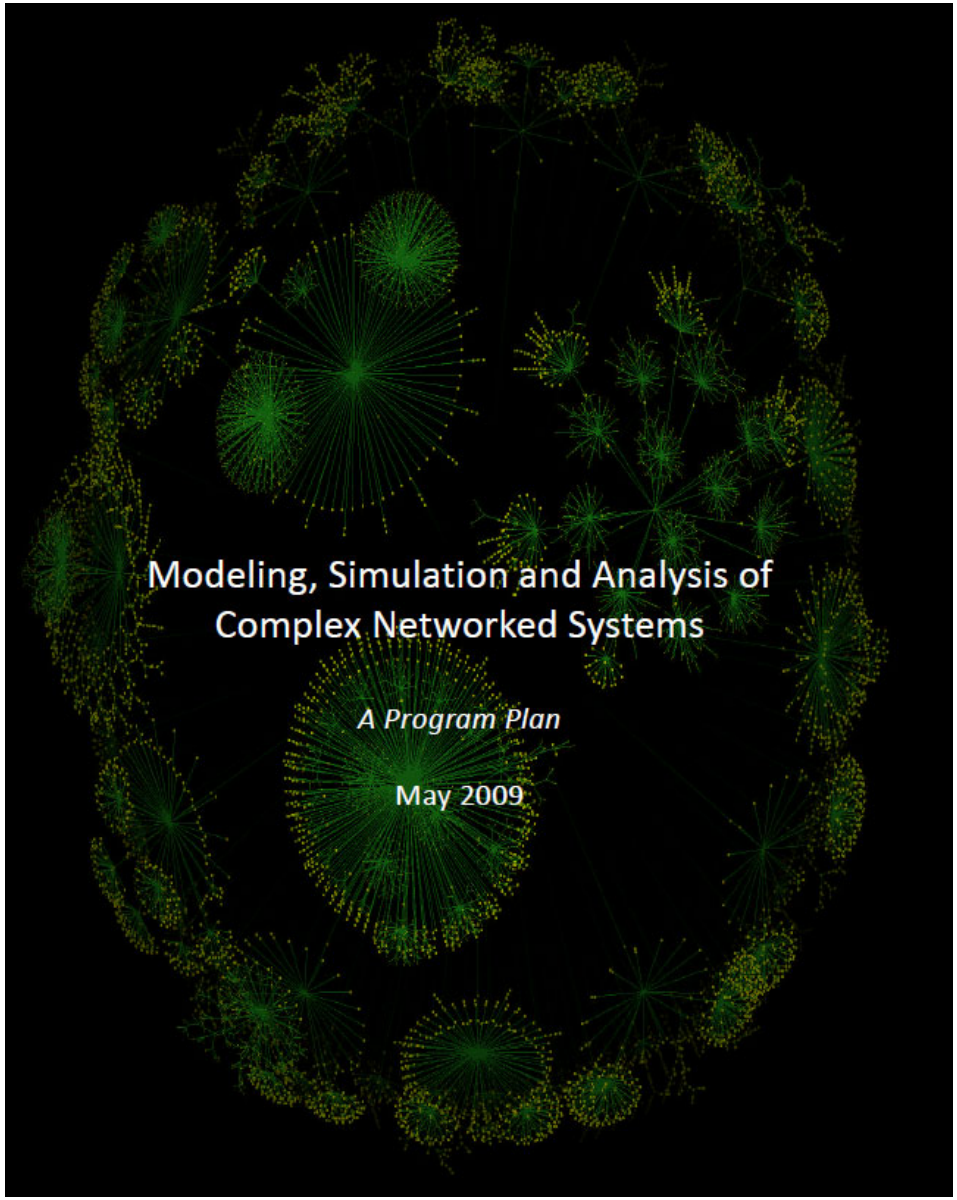
# Distribution of galaxies in the universe





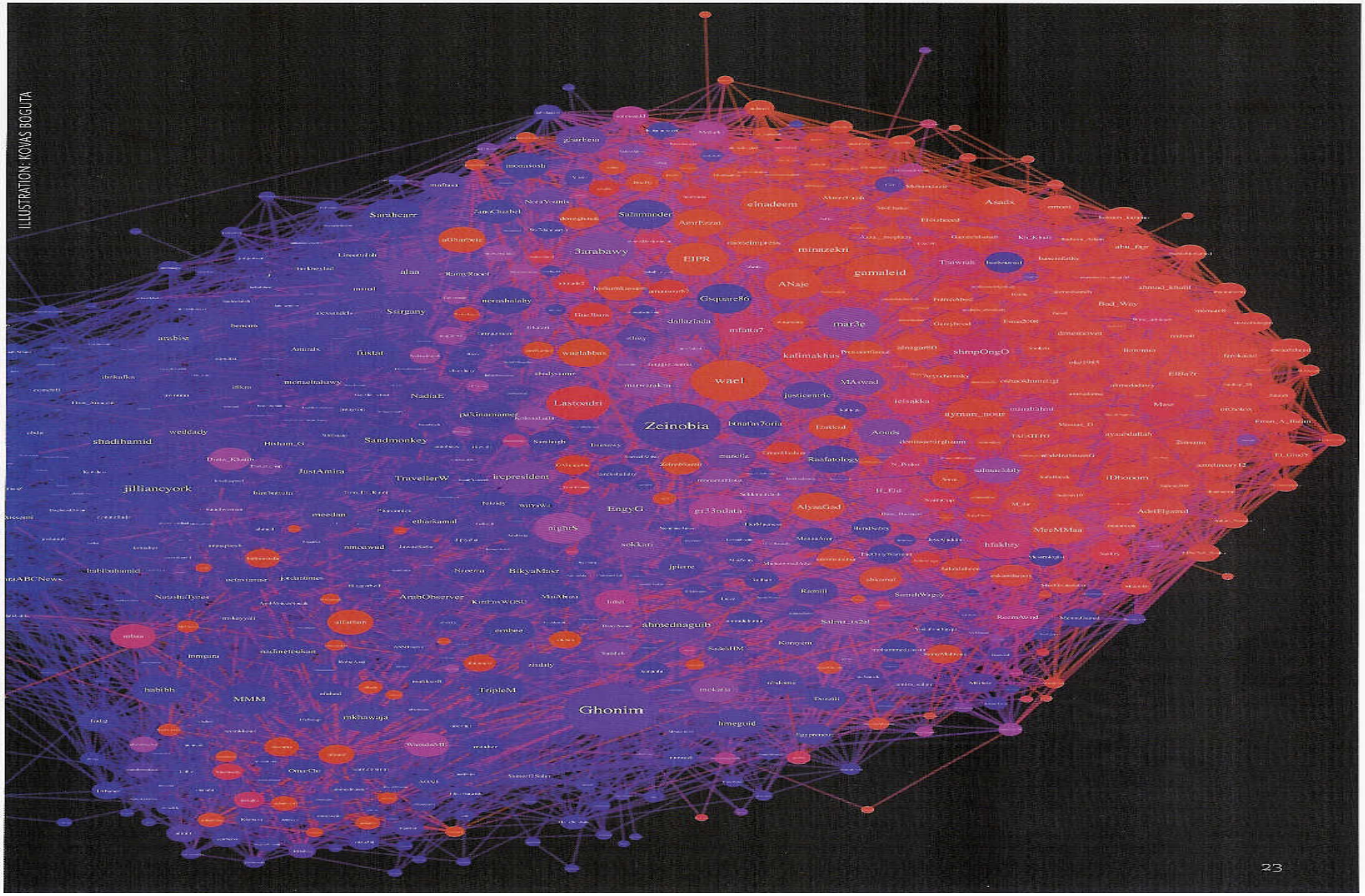




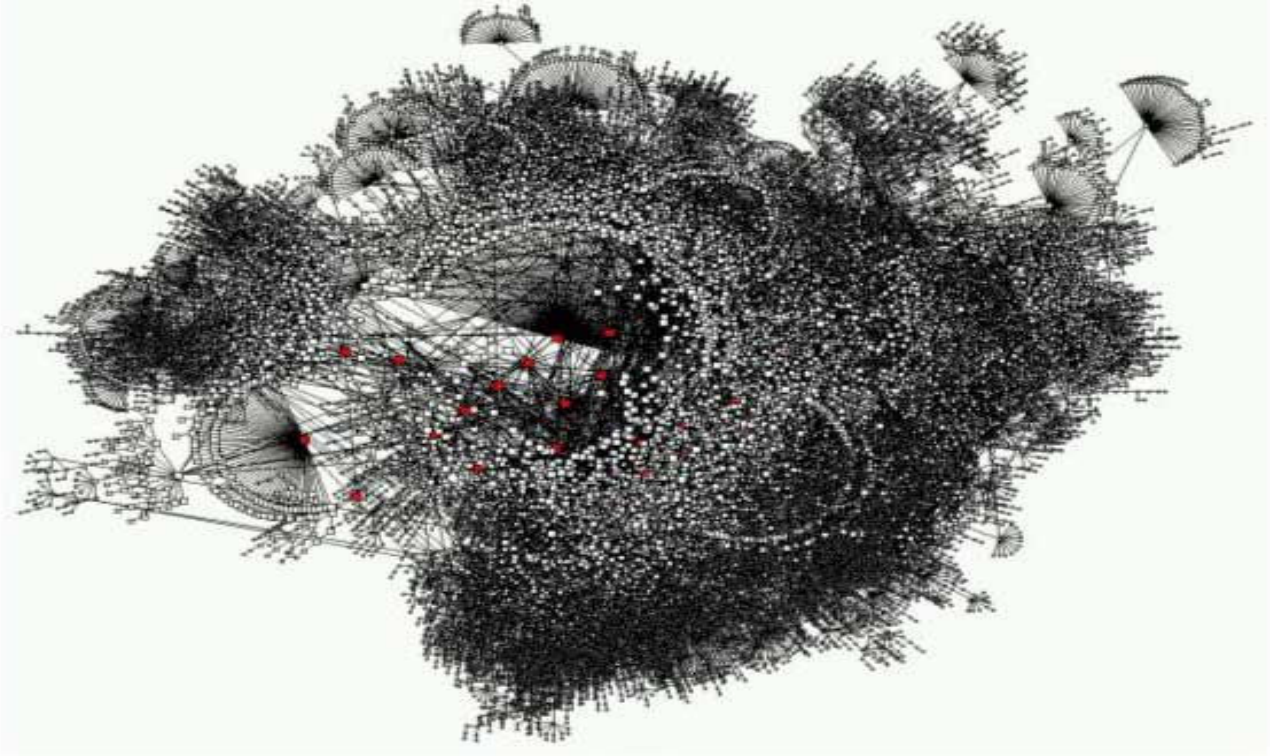



**Front and back covers:** Diagrams of the Abstract Syntax Tree (AST) internal representation of a simple C++ program. The firework burst patterns are namespaces and different parts of the internal classes, functions and code within functions. The front cover shows the AST for source code, while the back cover shows a similar AST used to represent the structure of binary executables for analysis.

**Source:** Dan Quinlan, Center for Applied Scientific Computing, Lawrence Livermore National Laboratory




The Egyptian Twitter Network
Arab users in Red, English users in purple



The internet

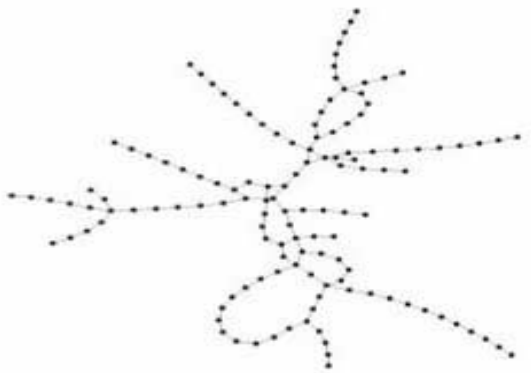
(a)



(b)



(c)



(d)

ecosystem

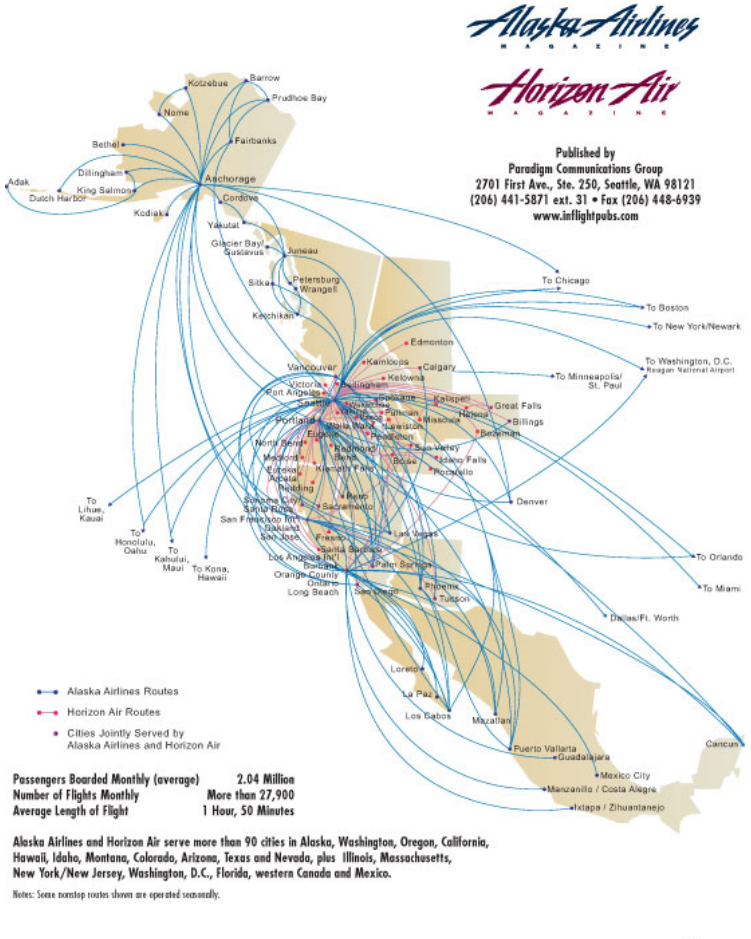
Professional
Collaboration network

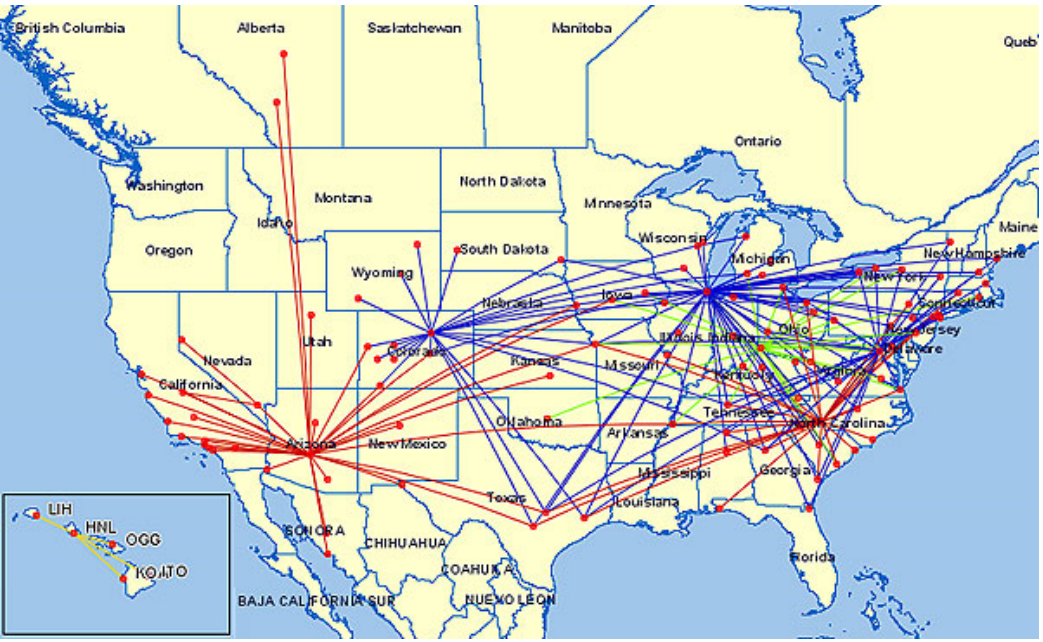
Barcelona rail network

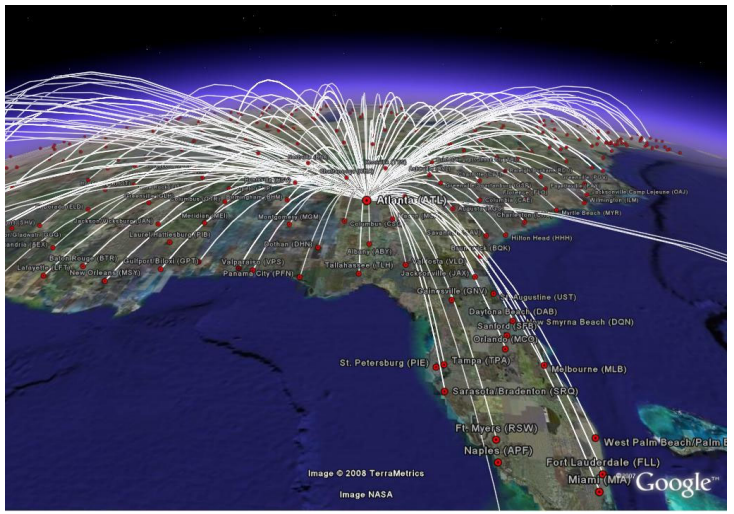
J.J. Finnigan, Complex science for a complex world

# Airline Route Systems



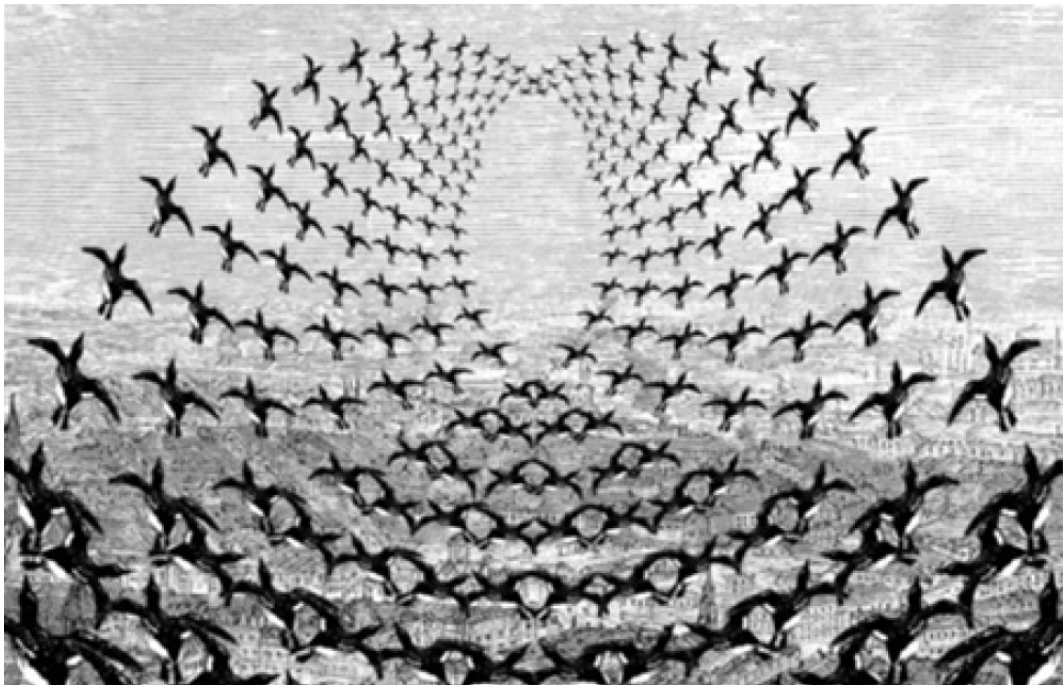



# 2. Motions of biological groups



Fish school

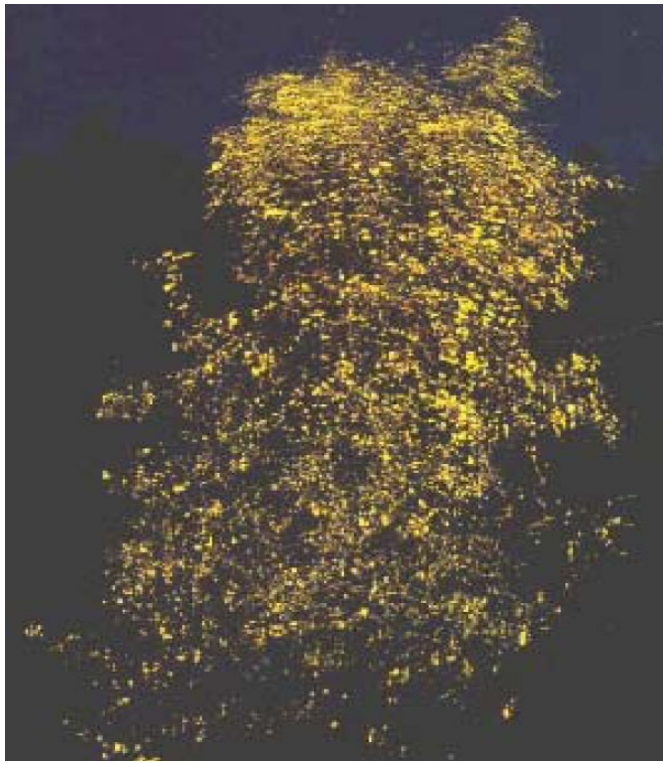
Birds flock





Locusts swarm

Fireflies synchronize



# Herd and Panic Behavior During Emergency Building Egress

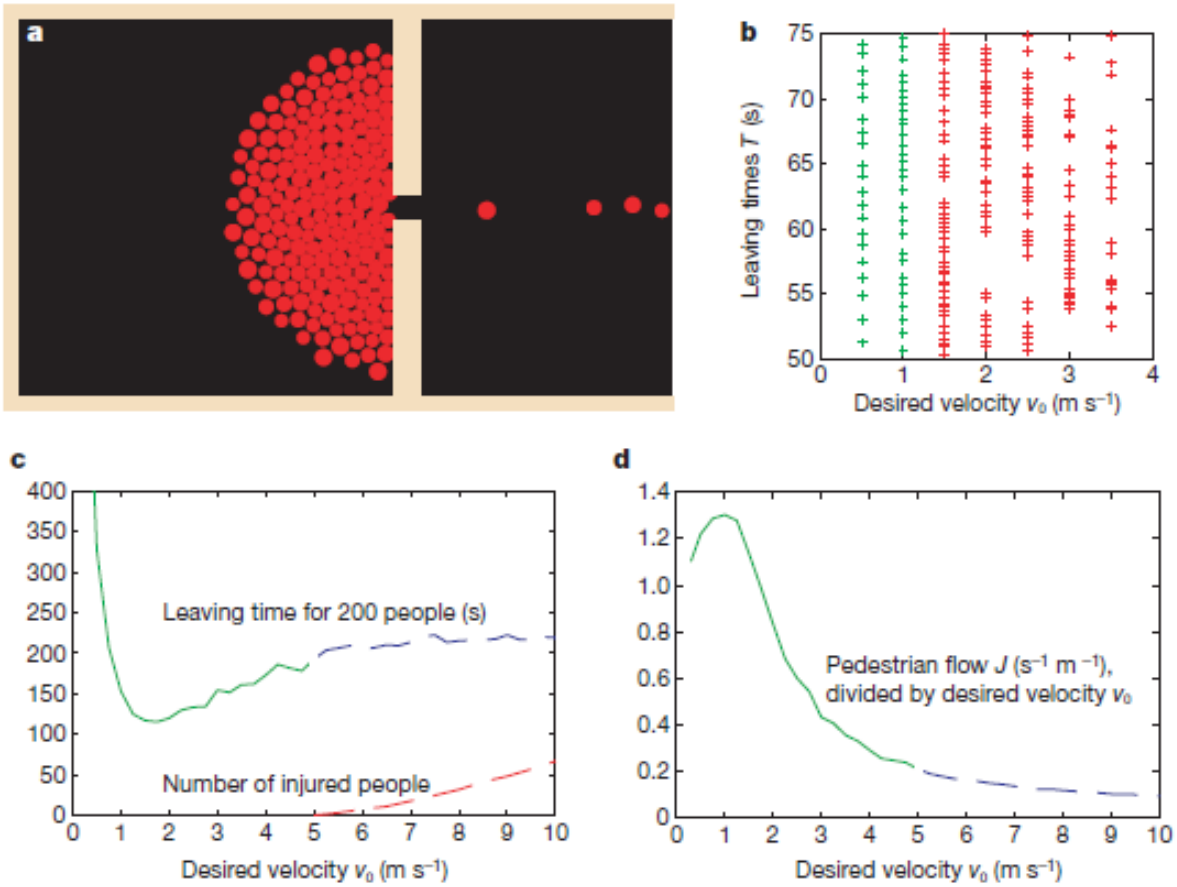
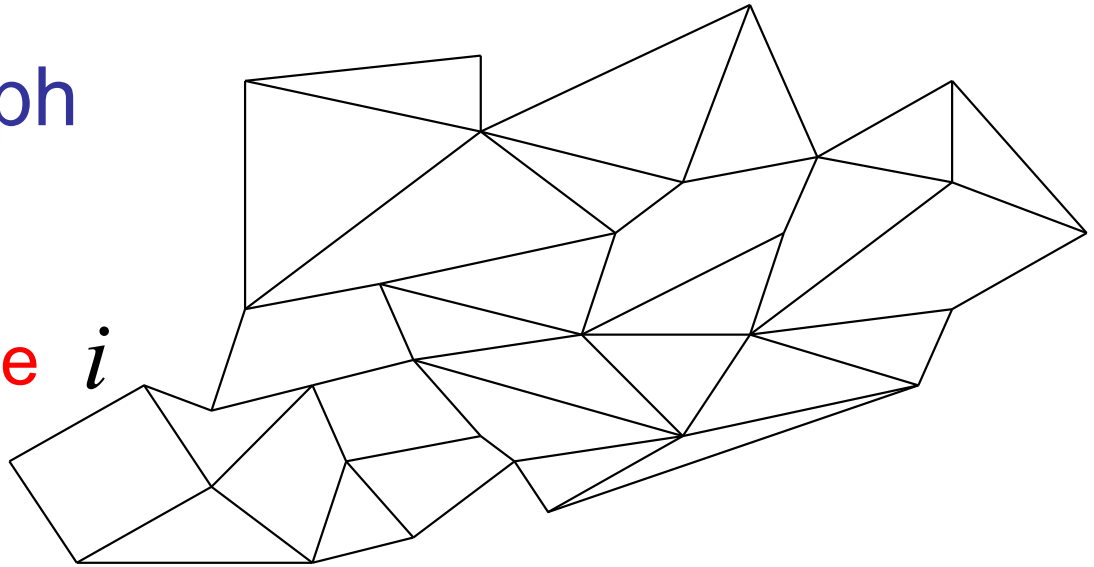



**Figure 1** Simulation of pedestrians moving with identical desired velocity $v_i^0 = v_0$ towards the 1-m-wide exit of a room of size 15 m × 15 m. **a**, Snapshot of the simulation. Dynamic simulations are available at http://angel.elte.hu/~panic/. **b**, Leaving times of pedestrians for various desired velocities $v_0$. Irregular outflow due to clogging is observed for high desired velocities ($v_0 \geq 1.5\,\mathrm{m\,s^{-1}}$, red plus signs). **c**, Under conditions of normal walking, the time for 200 pedestrians to leave the room decreases with growing $v_0$. Desired velocities higher than $1.5\,\mathrm{m\,s^{-1}}$ reduce the efficiency of leaving, which becomes particularly clear when the outflow $J$ is divided by the desired velocity (**d**). This is due to pushing, which causes additional friction effects. Moreover, above a desired velocity of about $v_0 = 5\,\mathrm{m\,s^{-1}}$ (corresponding to dashed lines in **c** and **d**) people are injured and become non-moving obstacles for others, if the sum of the magnitudes of the radial forces acting on them divided by their circumference exceeds a pressure of $1{,}600\,\mathrm{N\,m^{-1}}$ (ref. 5).

Owing to the above 'faster-is-slower effect', panics can be triggered by pedestrian counterflows[2], which cause delays to the crowd intending to leave. This makes the stopped pedestrians impatient and pushy which may be described by increasing the desired velocity according to $v_i^0(t) = [1 - p_i(t)]v_i^0(0) + p_i(t)v_i^{\max}$, where $v_i^0(0)$ is the initial, and $v_i^{\max}$ the maximum desired velocity. The time-dependent parameter $p_i(t) = 1 - \bar{v}_i(t)/v_i^0$, where $\bar{v}_i(t)$ denotes the average speed in the desired direction of motion, is a measure of impatience. Altogether, long waiting times increase the desired velocity, which can produce inefficient outflow. This further increases the waiting times, and so on, so that this tragic feedback can eventually trigger panics. It is therefore imperative to have sufficiently wide exits and to prevent counterflows when big crowds want to leave.

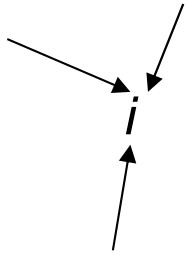
Helbring, Farkas, Vicsek, Nature 2000

# Communication Graph




N nodes (agents) interconnected by communication links.
Each agent can only get information from its neighbors.

$N_i$ Neighbors of node *i*




Each agent has dynamics $\dot{x}_i = Ax_i + Bu_i$

Study the interaction of control and communication

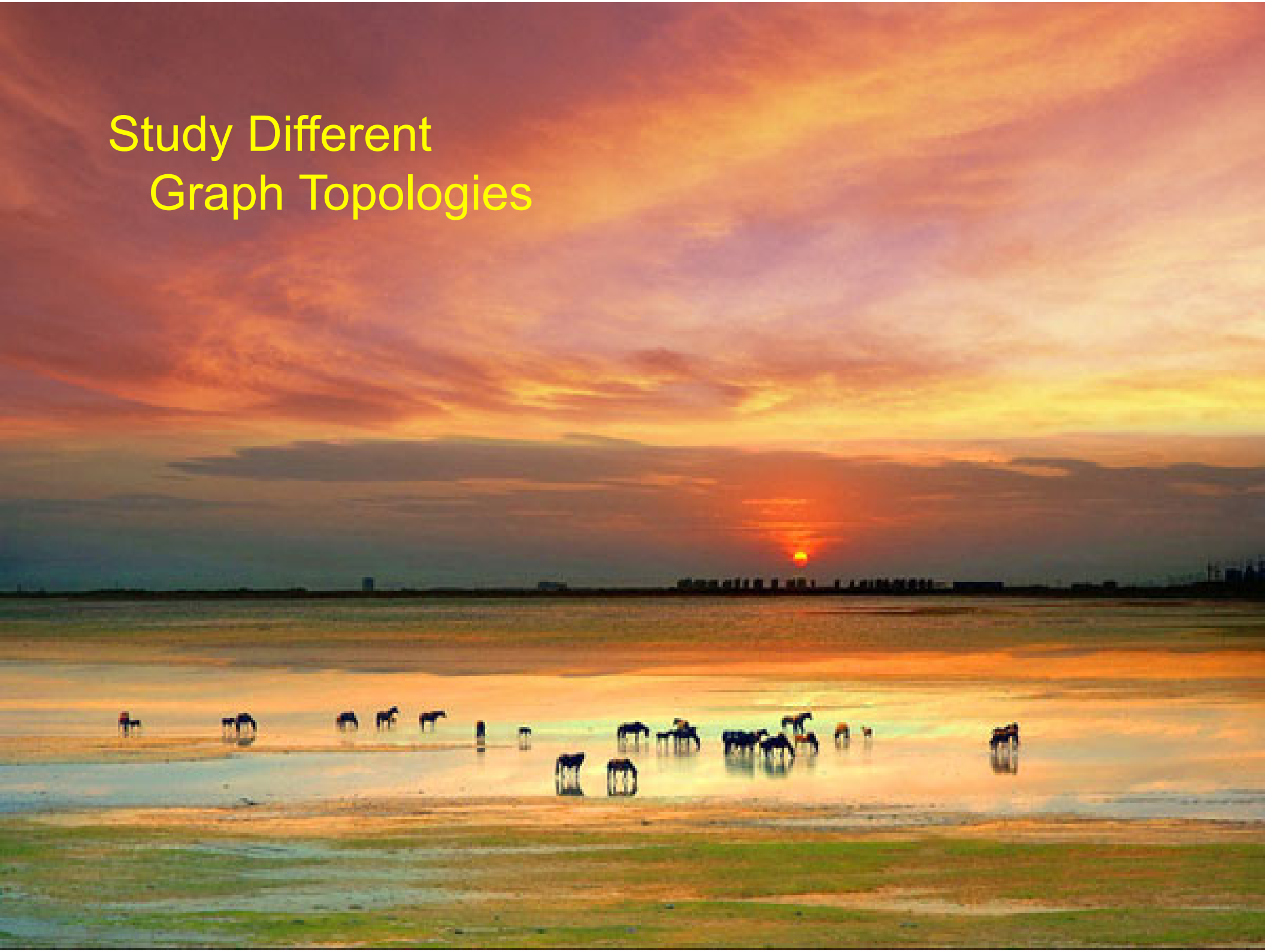


Birds communicate
Differently than Fish

So their Dynamical Motion
is Different

Study Different
Graph Topologies




Study Different
Graph Topologies

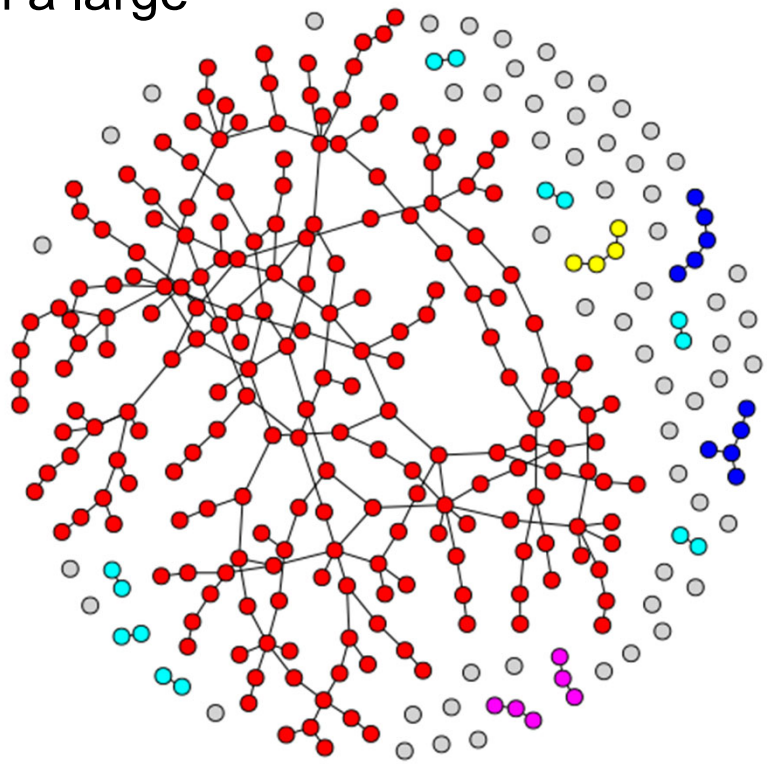
# 1. Random Graphs – Erdos and Renyi

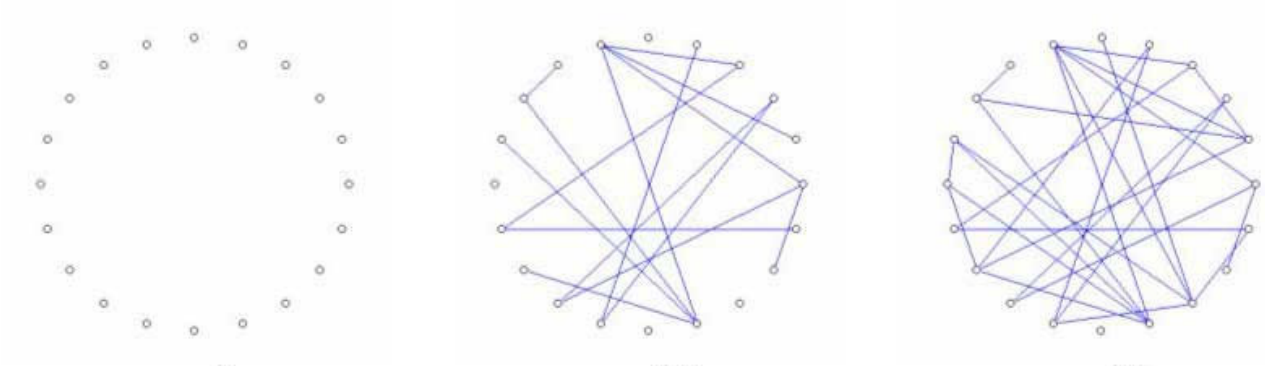
N nodes
Two nodes are connected with probability p independent of other edges

## Phase Transition

m= number of edges

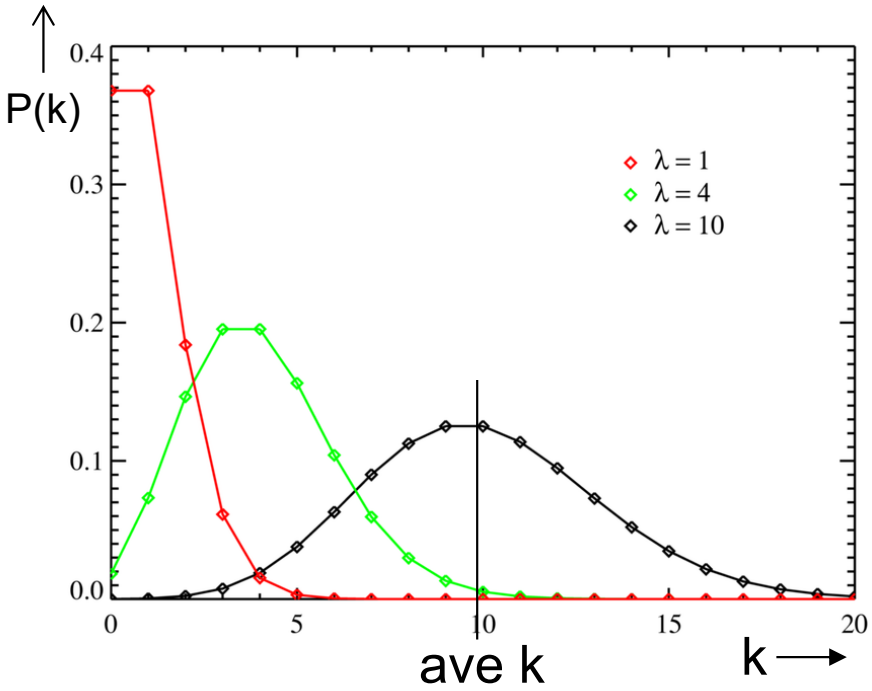
There is a critical threshold $m_0(n) = N/2$ above which a large connected component appears – giant clusters






J.J. Finnigan, Complex science for a complex world

Connectivity- degree distribution is Poisson
Homogeneity- all nodes have about the same degree



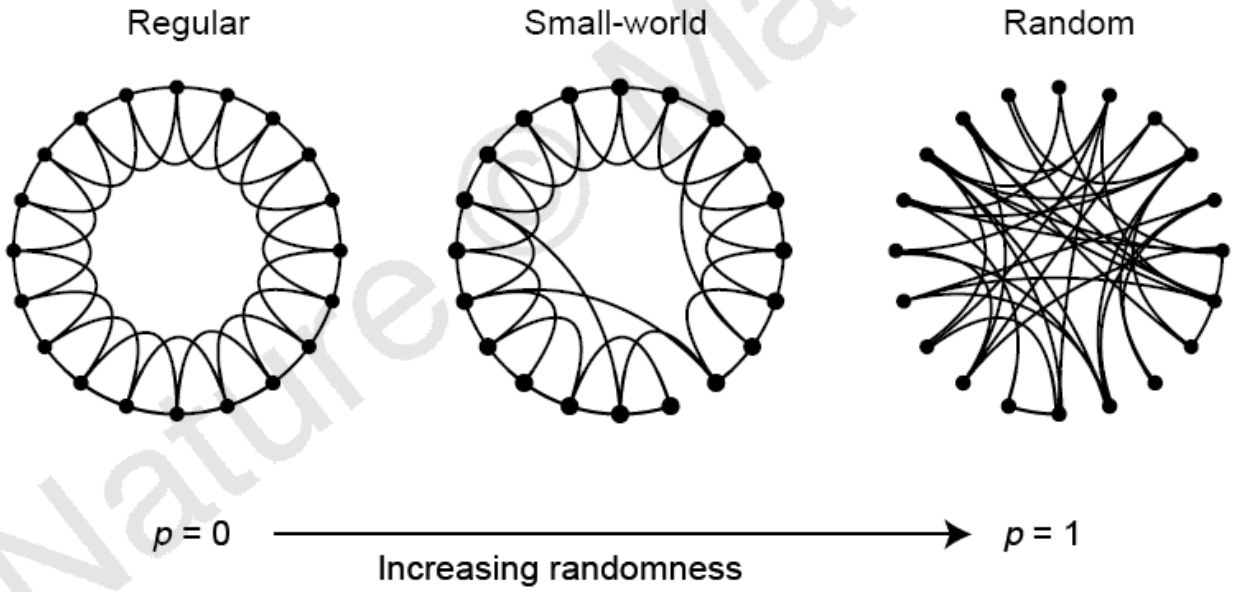

Poisson degree distribution
most nodes have about the same degree
ave(k) depends on number of nodes

# 2. Small World Networks- Watts and Strogatz

Start with a regular lattice
With probability p, rewire an edge to a random node.

Connectivity- degree distribution is Poisson
Homogeneity – all nodes have about the same degree

Small diameter (longest path length)
Large clustering coeff.- i.e. neighbors are connected




Nature © Mac

Watts & Strogatz, Nature 1998

# Phase Transition
# Diameter and Clustering Coeficient

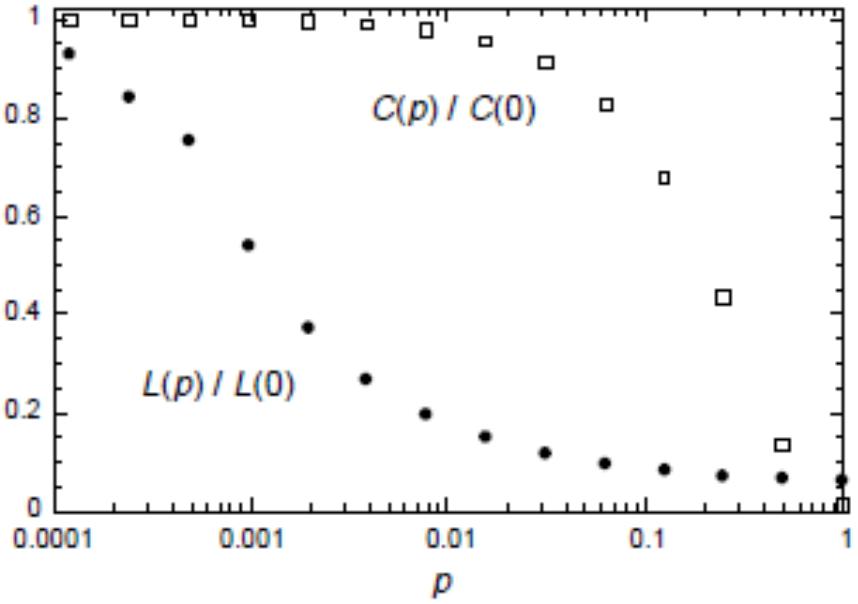
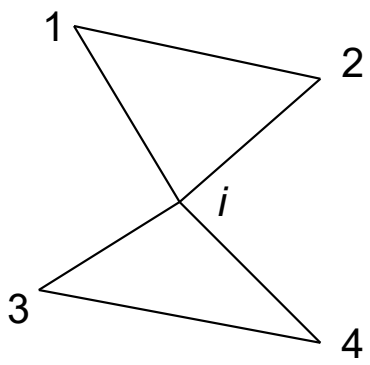


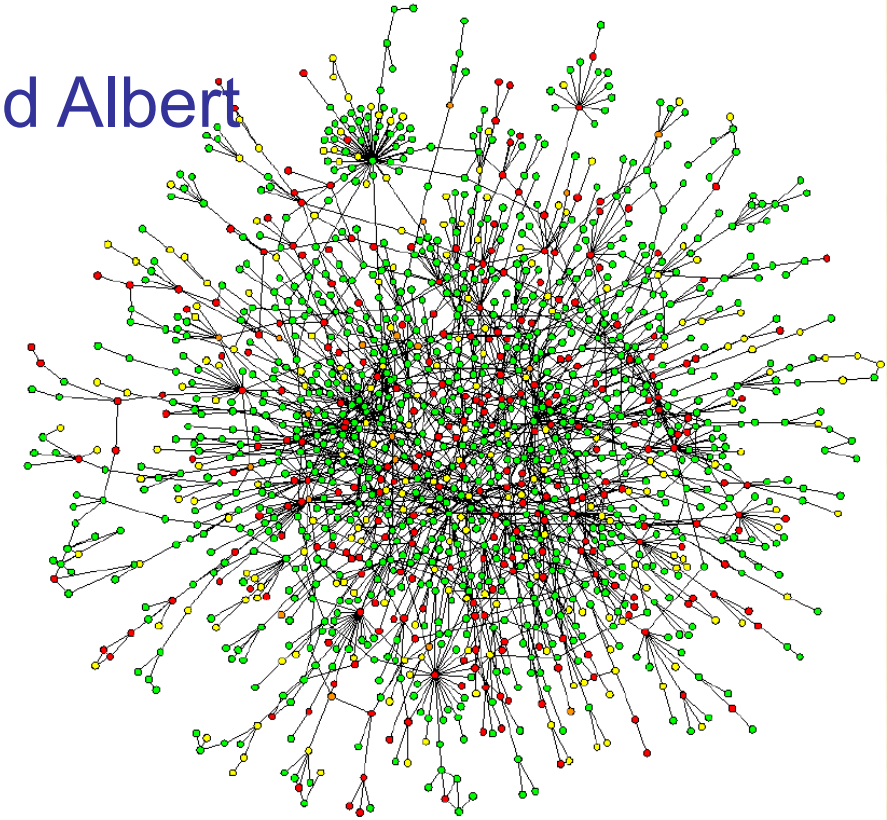

**Figure 2** Characteristic path length $L(p)$ and clustering coefficient $C(p)$ for the family of randomly rewired graphs described in Fig. 1. Here $L$ is defined as the number of edges in the shortest path between two vertices, averaged over all pairs of vertices. The clustering coefficient $C(p)$ is defined as follows. Suppose that a vertex $v$ has $k_v$ neighbours; then at most $k_v(k_v-1)/2$ edges can exist between them (this occurs when every neighbour of $v$ is connected to every other neighbour of $v$). Let $C_v$ denote the fraction of these allowable edges that actually exist. Define $C$ as the average of $C_v$ over all $v$. For friendship networks, these

## Clustering coefficient




Nr of neighbors of i = 4
Max nr of nbr interconnections= 4x3/2= 6
Actual nr of nbr interconnects= 2
Clustering coeff= 2/6= 1/3

# 3. Scale-Free Networks– Barabasi and Albert

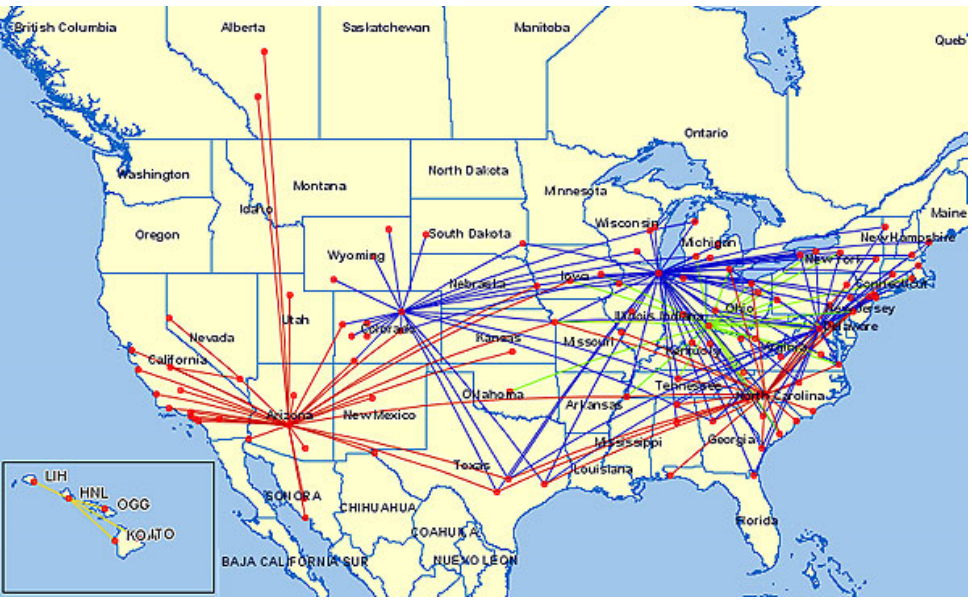


Start with $m_0$ nodes
Add one node at a time:
connect to m other nodes

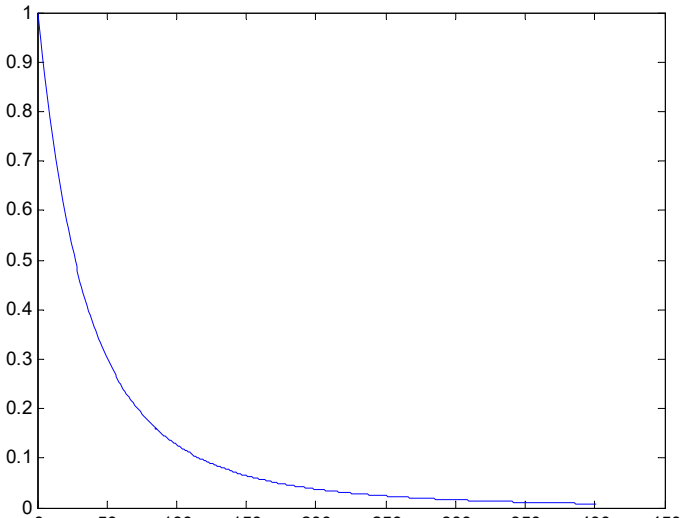
with probability $P(i) = \frac{d_i + 1}{\sum_j (d_j + 1)}$

i.e. with highest probability to biggest nodes
(rich get richer)

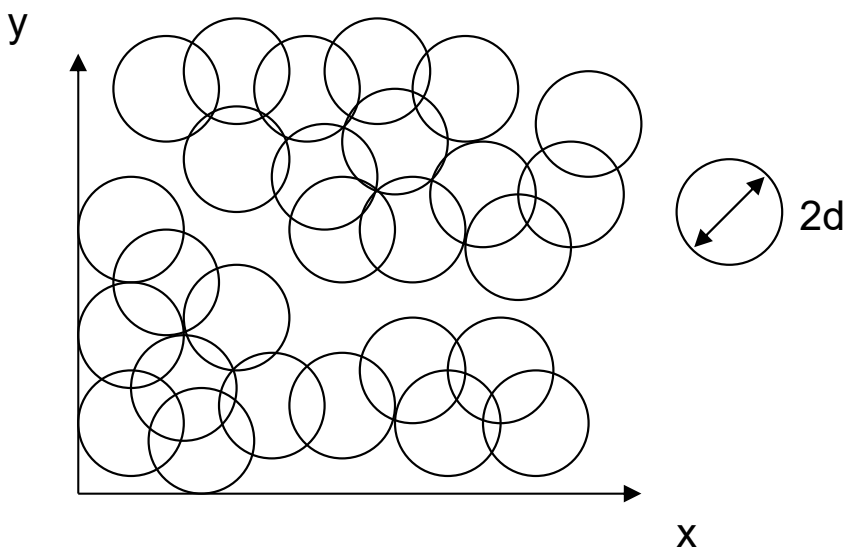
Nonhomogeneous- some nodes have large degree, most have small degree
Scale-Free- degree has power law degree distribution




$$P(k) = \frac{2m^2}{k^3}$$



# 4. Proximity Graphs




Randomly select N points in the plane
Draw an edge (i,j) if distance between nodes i and j is within d

When is the graph connected?
for what values of (N,d)
What is the degree distribution?

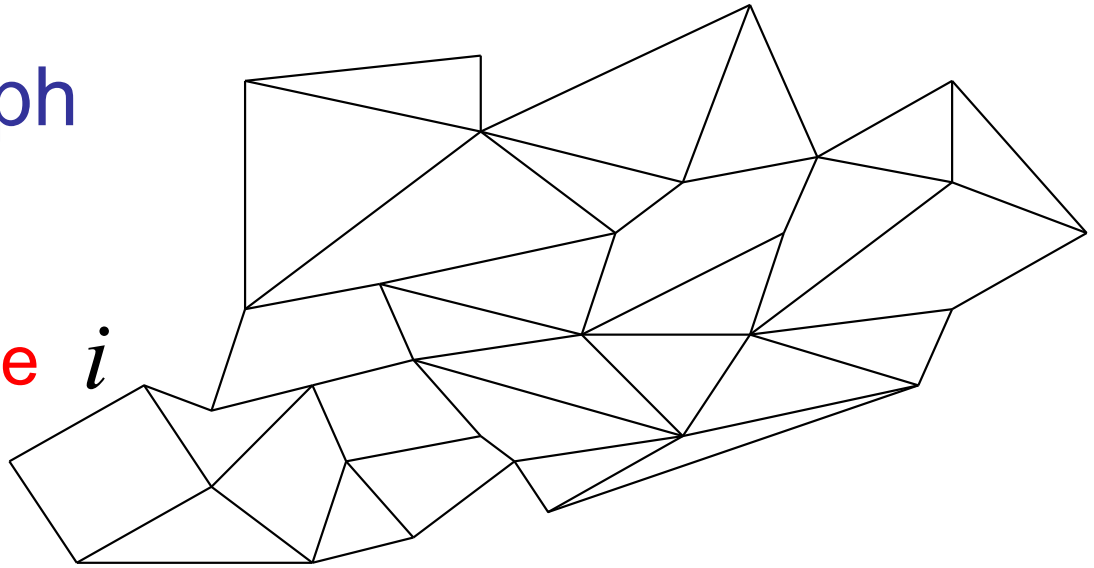
## Mobile Sensor Networks Project with Singapore A-Star

# The Interaction Between Communication and Control

**The way we communicate**
**decides the way we evolve dynamically**



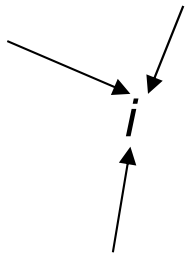
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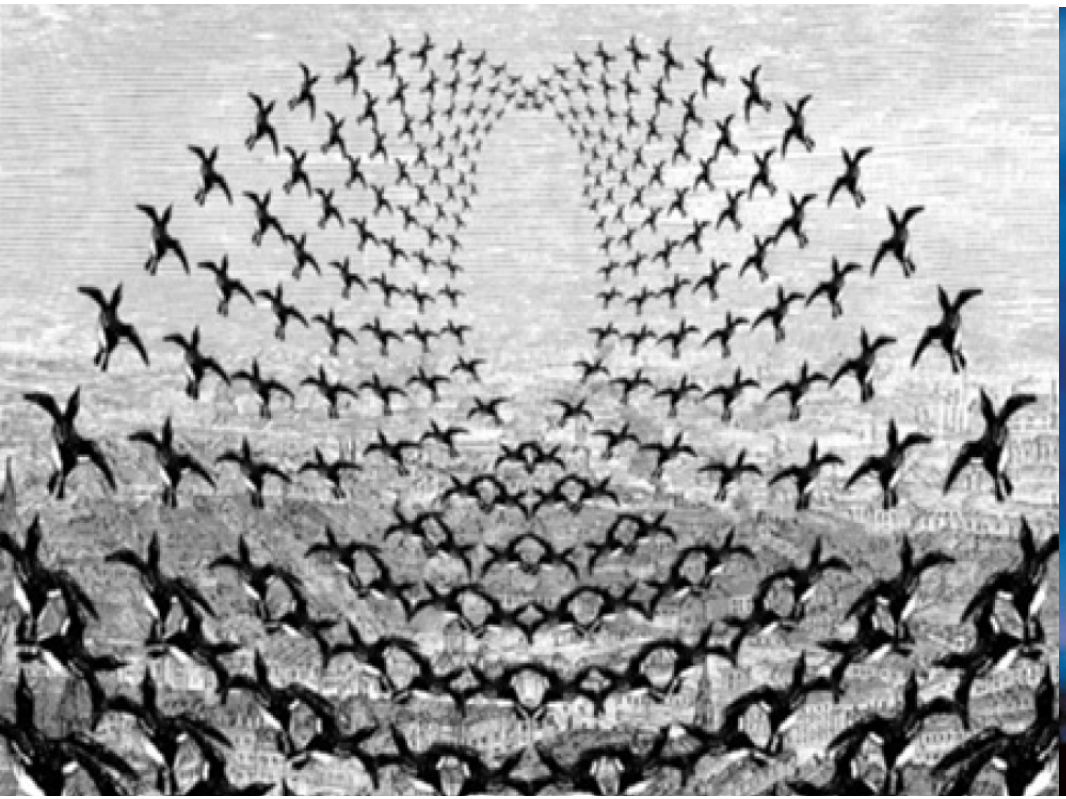
Study the interaction of control and communication

Reynolds, Computer Graphics 1987

# Flocking

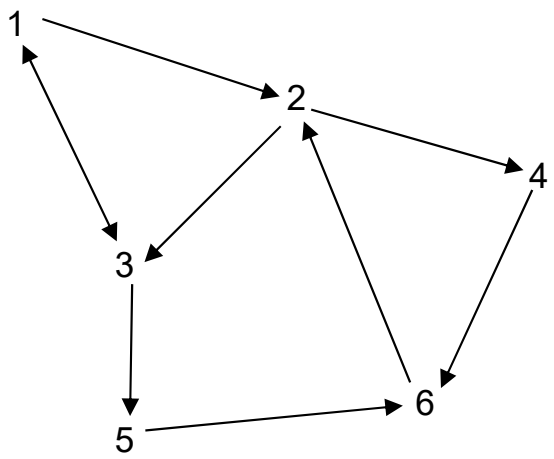
## Reynolds' Rules:

1. Alignment : align headings $\dot{\theta}_i = \sum_{j \in N_i} a_{ij}(\theta_j - \theta_i)$
2. Cohesion : steer towards average position of neighbors- towards c.g.
3. Separation : steer to maintain separation from neighbors





# Consensus Control for Swarm Motions

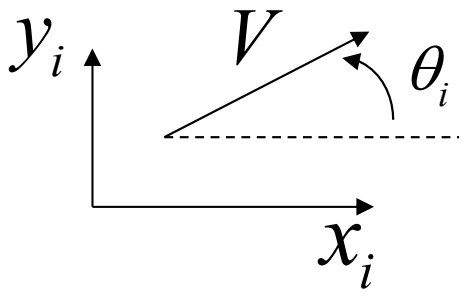


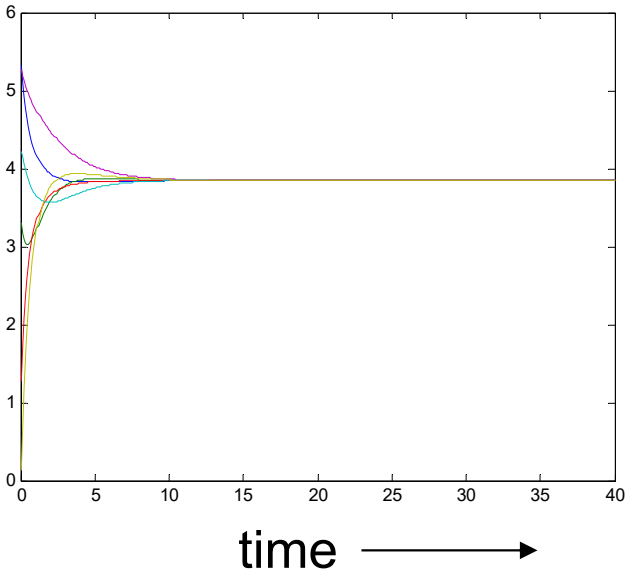

$$\dot{\theta}_i = \sum_{j \in N_i^c} a_{ij}(\theta_j - \theta_i)$$

Heading angle

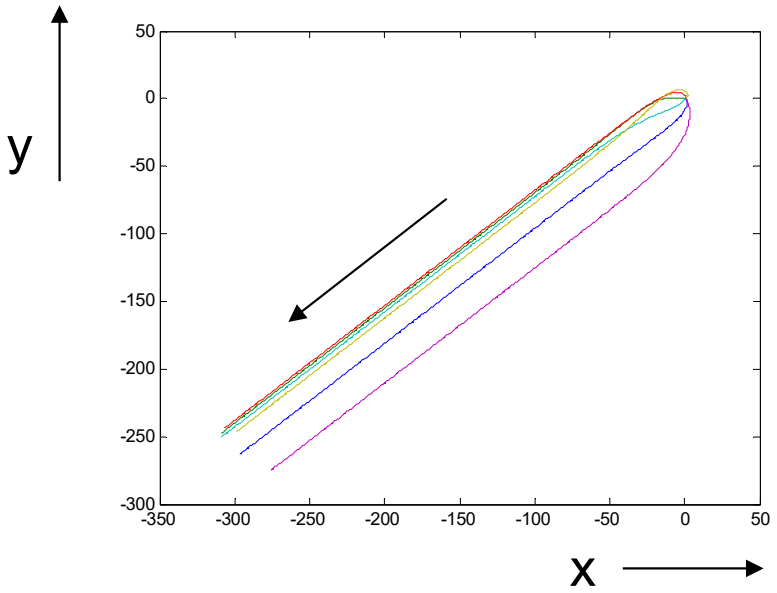
$$\dot{x}_i = V\cos\theta_i$$
$$\dot{y}_i = V\sin\theta_i$$

Motion Dynamics



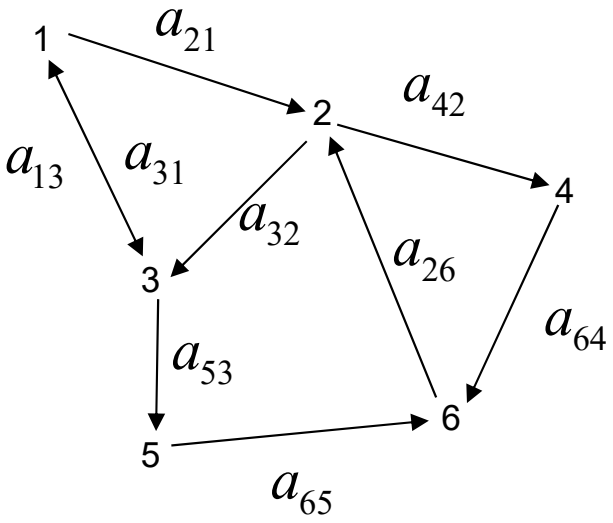




Convergence of headings




Nodes converge to consensus heading

# Consensus Control for Swarm Motions




$$\dot{\theta}_i = \sum_{j \in N_i^c} a_{ij}(\theta_j - \theta_i)$$

heading angle

Edge weight $i \xleftarrow{a_{ij}} j$

$$\dot{\theta}_1 = a_{13}(\theta_3 - \theta_1)$$

$$\dot{\theta}_2 = a_{21}(\theta_1 - \theta_2) + a_{26}(\theta_6 - \theta_2)$$

$$\dot{\theta}_3 = a_{31}(\theta_1 - \theta_3) + a_{32}(\theta_2 - \theta_3)$$

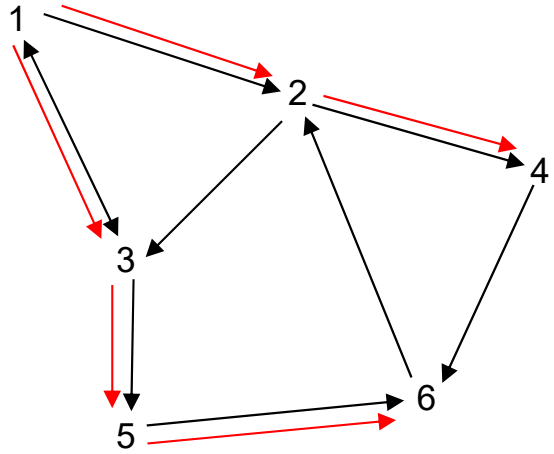
$$\dot{\theta}_4 = a_{42}(\theta_2 - \theta_4)$$

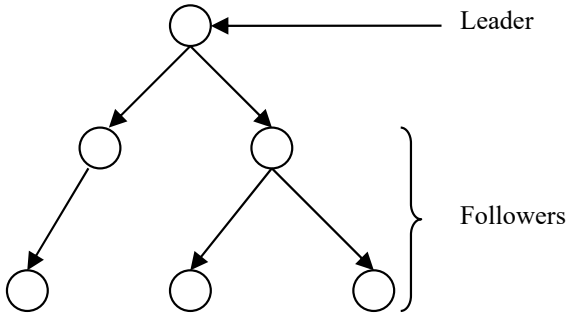
$$\dot{\theta}_5 = a_{53}(\theta_3 - \theta_5)$$

$$\dot{\theta}_6 = a_{65}(\theta_5 - \theta_6) + a_{64}(\theta_4 - \theta_6)$$

Alignment of Headings

# Connected graph




**Definition.** Graph contains a **spanning tree** if there is a path from one node to all the rest.

Then the graph is **connected**

**Theorem :** Let graph have a spanning tree.
Then all nodes reach the same heading.



Birds communicate
Differently than Fish

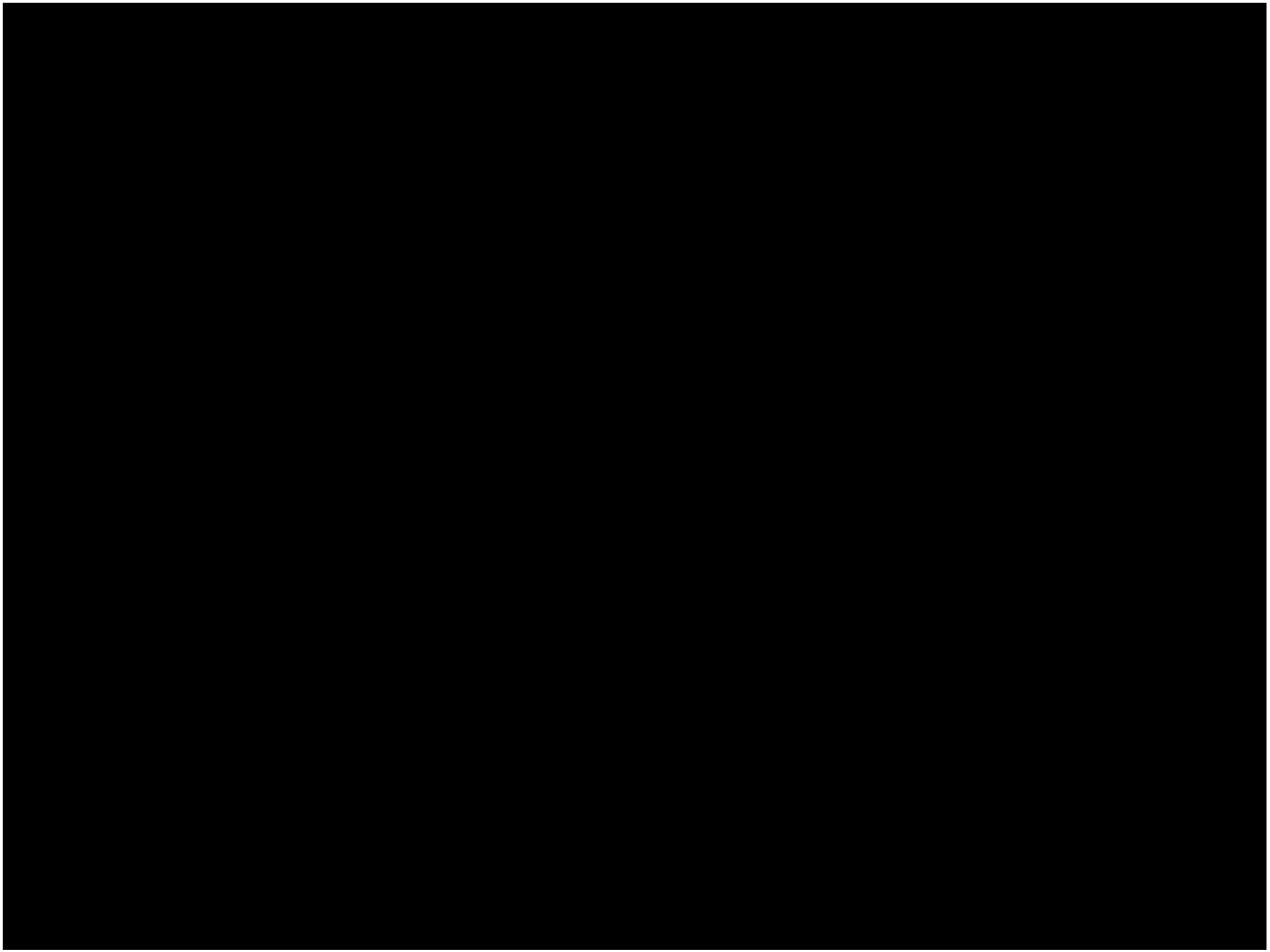
So their Dynamical Motion
is Different

Behavior on Different
Graph Topologies



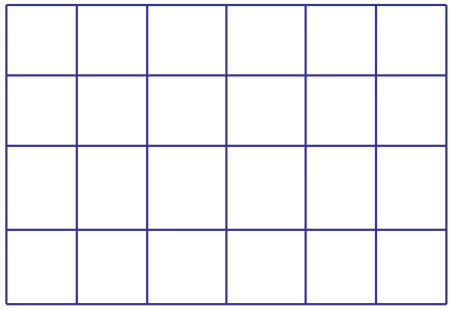
# The Power of Synchronization

## Coupled Oscillators
## Diurnal Rhythm

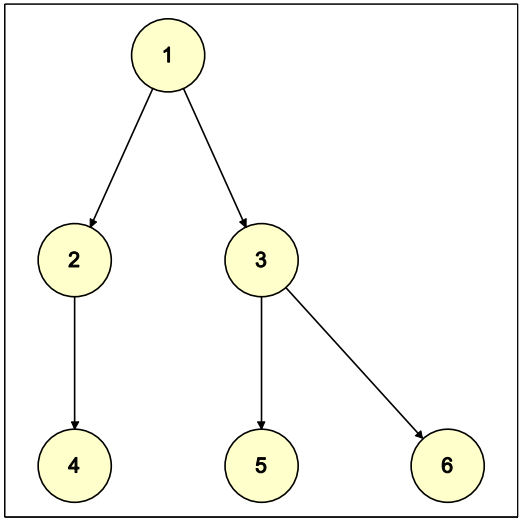


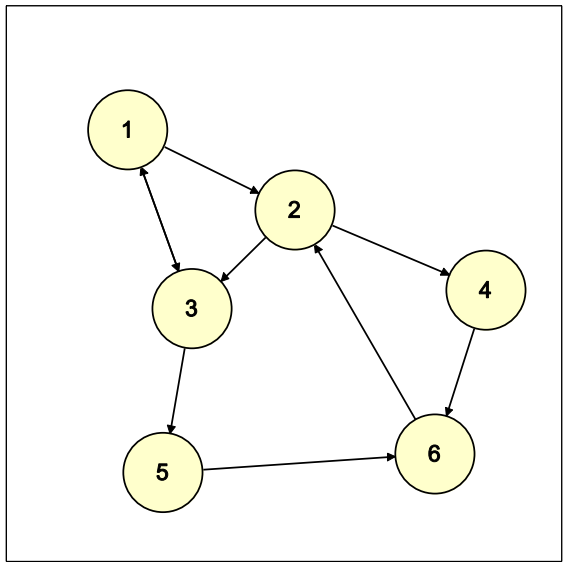
# Synchronization on Good Graphs

Chris Elliott fast video



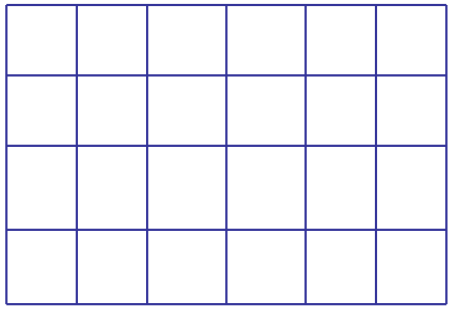
Regular mesh

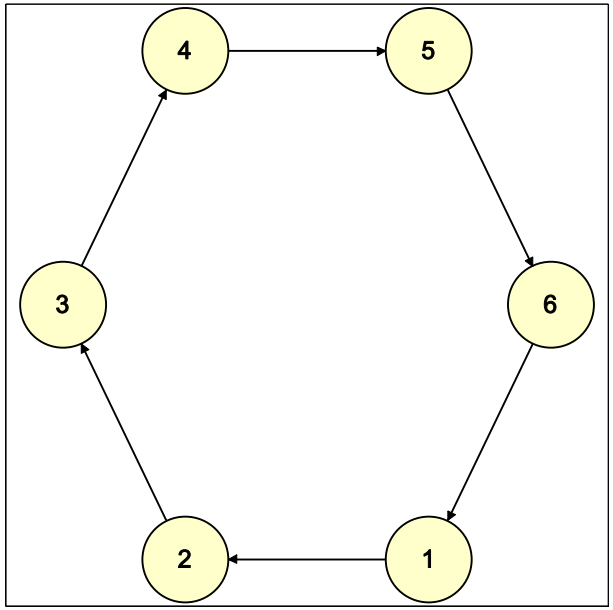


# Synchronization on Gossip Rings

Chris Elliott weird video







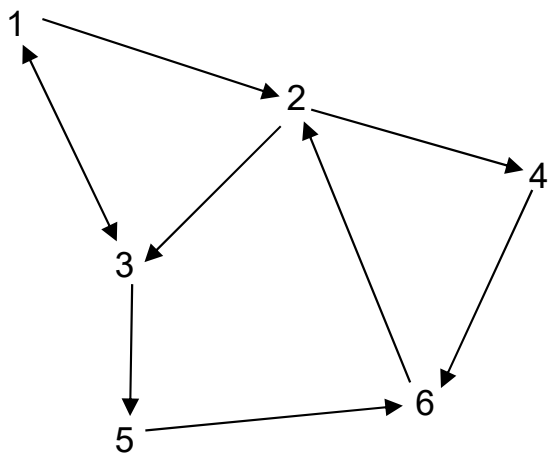
# Distributed Adaptive Control for Multi-Agent Systems








# Consensus Control for Swarm Motions

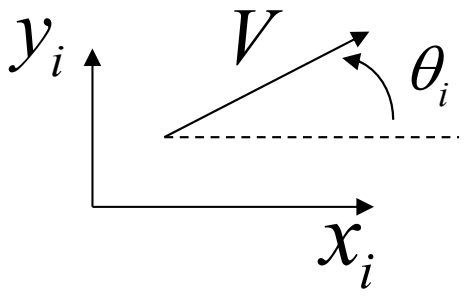


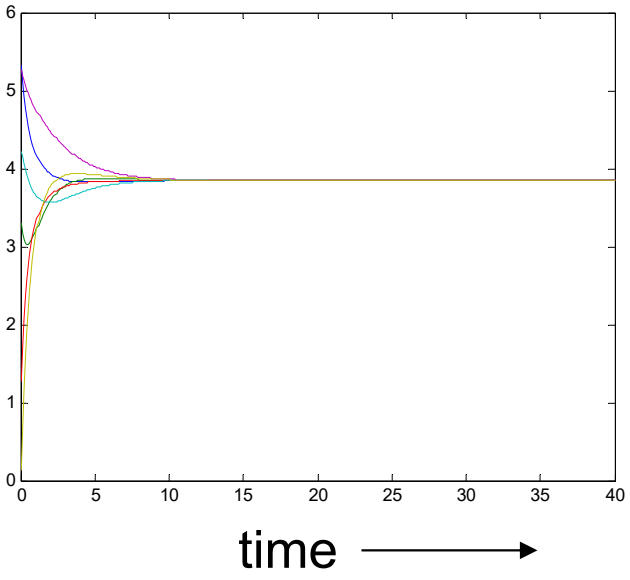

$$\dot{\theta}_i = \sum_{j \in N_i^c} a_{ij}(\theta_j - \theta_i)$$

Heading angle

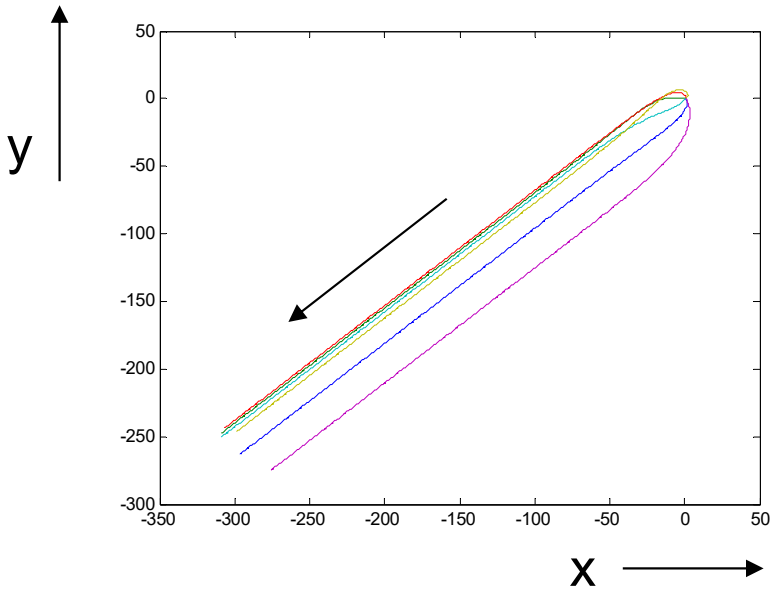
$$\dot{x}_i = V\cos\theta_i$$
$$\dot{y}_i = V\sin\theta_i$$

Motion Dynamics



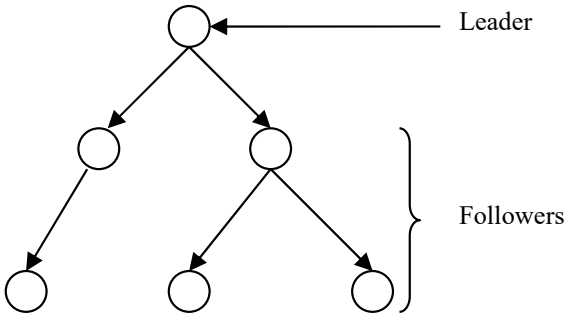




Convergence of headings




Nodes converge to consensus heading

# Consensus Control for Formations

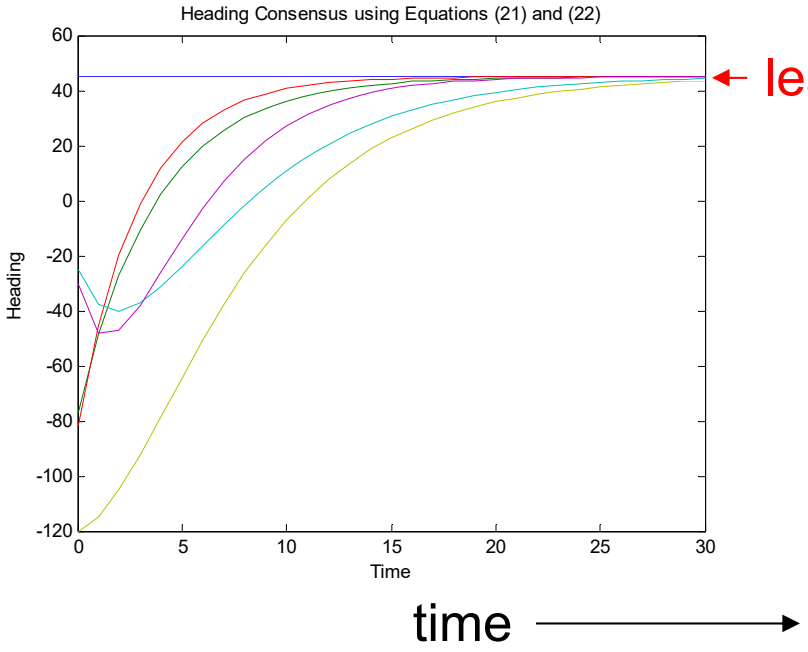



$$\dot{\theta}_i = \sum_{j \in N_i^c} a_{ij}(\theta_j - \theta_i)$$ heading angle

$$\dot{x}_i = V\cos\theta_i$$

$$\dot{y}_i = V\sin\theta_i$$

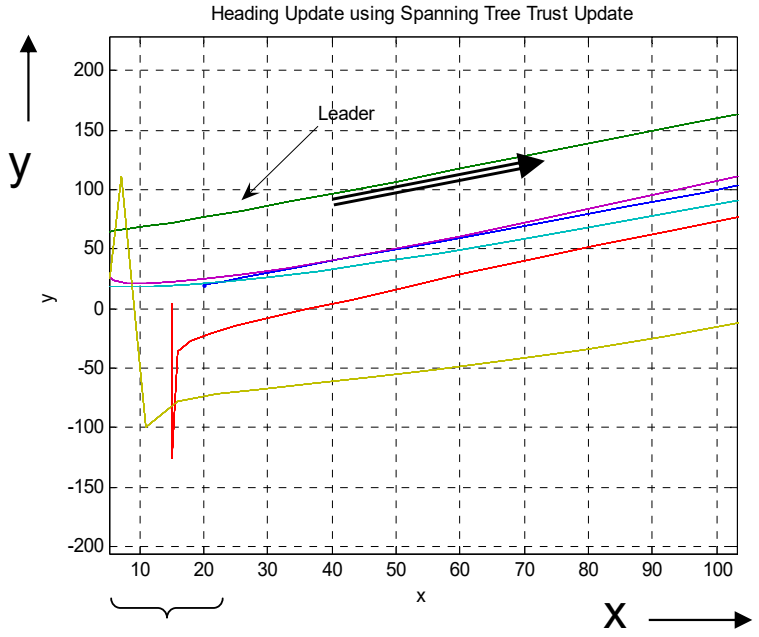
## Formation- a Tree network




← leader

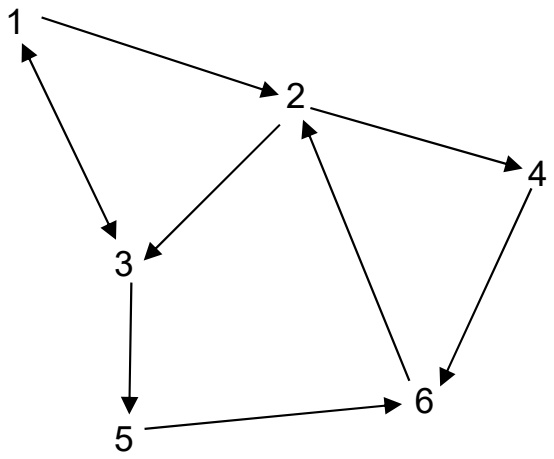
time →

Convergence of headings




Nodes converge to heading of leader

# Trust-Based Control: Swarms/Formations

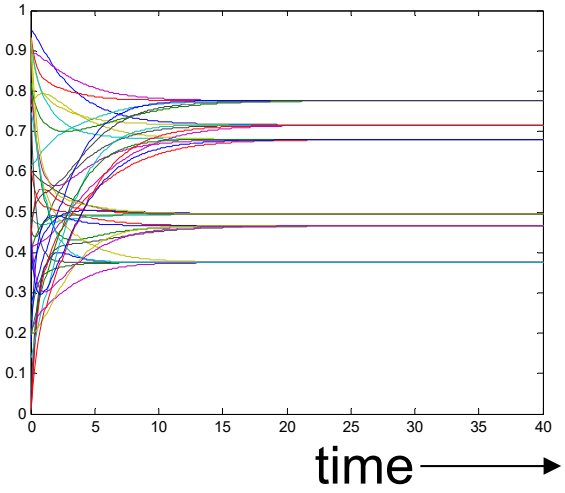



Trust dynamics $\dot{\xi}_i = \sum_{j \in N_i} a_{ij}(\xi_j - \xi_i)$

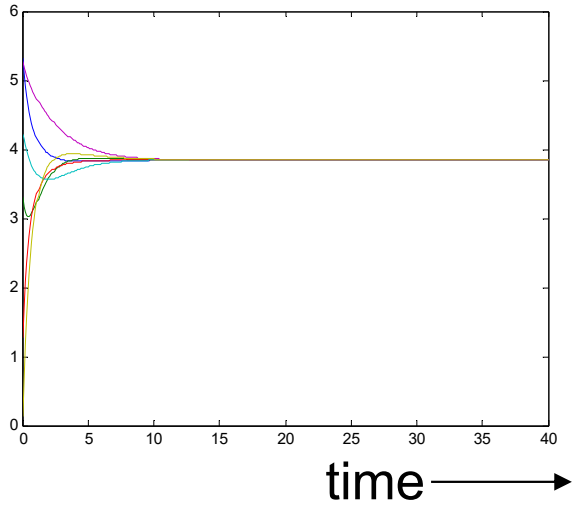
Motion dynamics $\dot{\theta}_i = \sum_{j \in N_i^c} \xi_{ij} a_{ij}(\theta_j - \theta_i)$ heading angle

$\dot{x}_i = V \cos\theta_i$

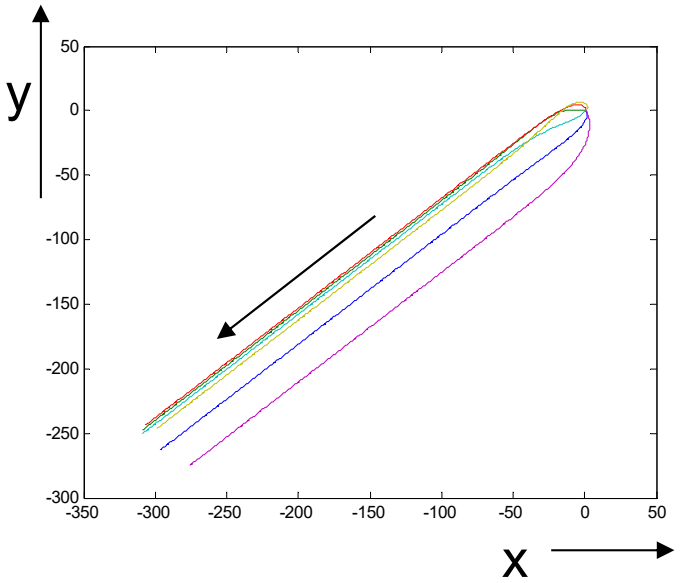
$\dot{y}_i = V \sin\theta_i$




Convergence of trust




Convergence of headings




Nodes converge to consensus heading

# Trust Propagation and Consensus – Network Security

Inspired by social behavior in flocks, herds, teams

Foundation work by John Baras

Define $\xi_{ij}$ as the trust that node i has for node j

$\xi_{ij} \in [-1,1]$

-1 ..... 0 ..... 1

Distrust no opinion complete trust

Define trust vector of node i as

$$\xi_i = \begin{bmatrix} \xi_{i1} \\ \xi_{i2} \\ \xi_{i3} \\ \xi_{i4} \\ \xi_{i5} \\ \xi_{i6} \end{bmatrix}$$

Trust node i has for node 3

N vector

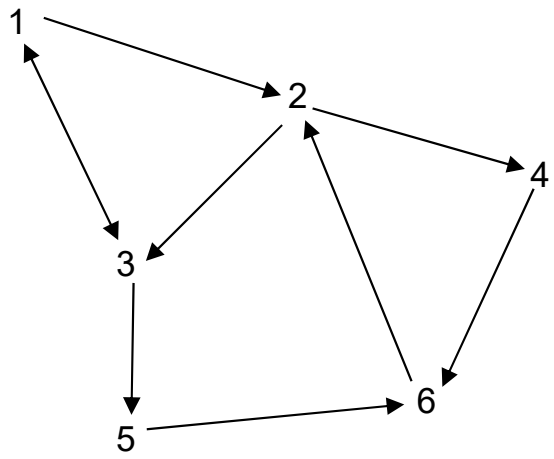
$$\xi \equiv \begin{bmatrix} \xi_1 \\ \xi_2 \\ \vdots \\ \xi_N \end{bmatrix} \in R^{N^2}$$

Standard local voting protocol $\dot{\xi}_i = u_i = \sum_{j \in N_i} a_{ij} (\xi_j - \xi_i)$

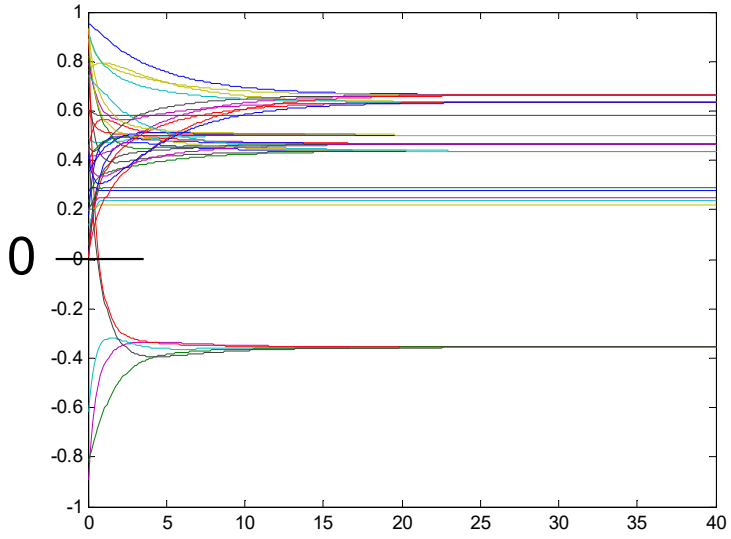
Difference of opinion with neighbors

Closed-loop trust dynamics $\dot{\xi} = -(L \otimes I_N)\xi$

# Trust Propagation & Consensus



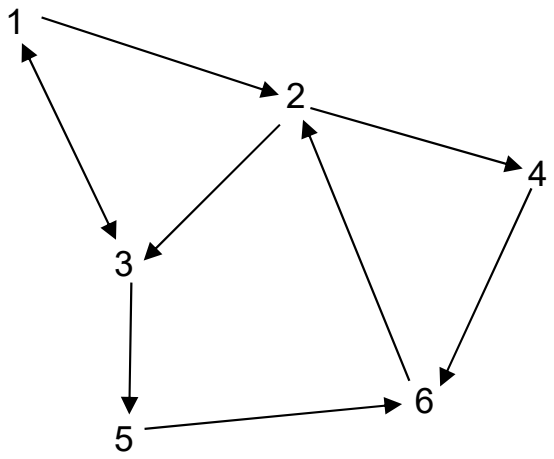

Nodes 1, 2, 4 initially distrust node 5
initial trusts are negative




Other nodes agree that
node 5 has negative trust

Convergence of trust

# Trust-Based Control: Swarms/Formations

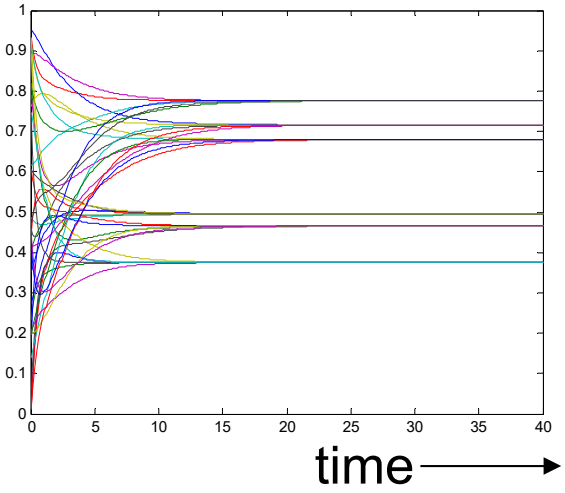



Trust dynamics $\dot{\xi}_i = \sum_{j \in N_i} a_{ij}(\xi_j - \xi_i)$

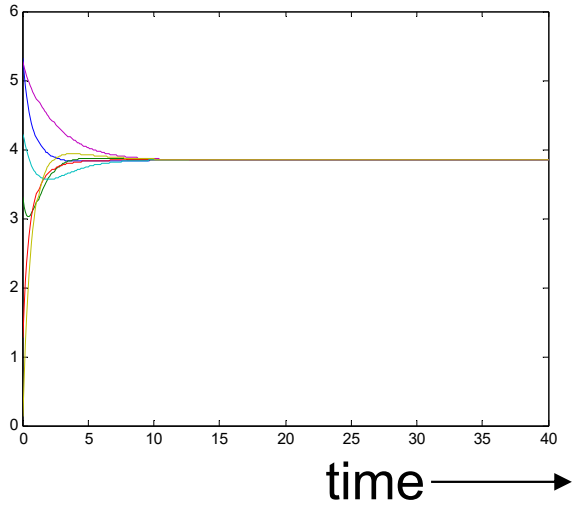
Motion dynamics $\dot{\theta}_i = \sum_{j \in N_i^c} \xi_{ij} a_{ij}(\theta_j - \theta_i)$ heading angle

$\dot{x}_i = V \cos\theta_i$

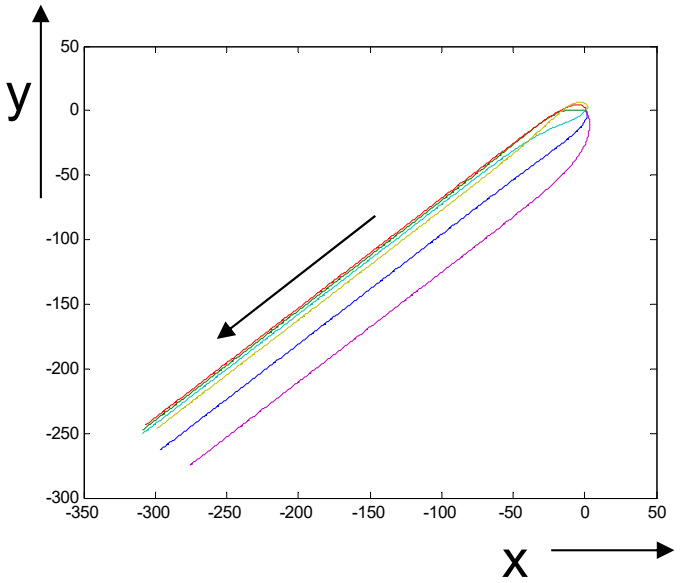
$\dot{y}_i = V \sin\theta_i$




Convergence of trust




Convergence of headings

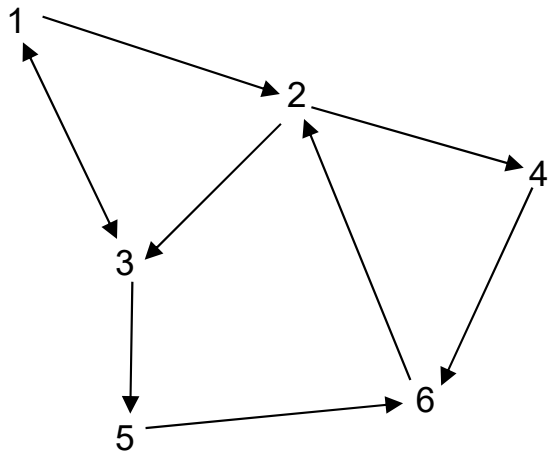


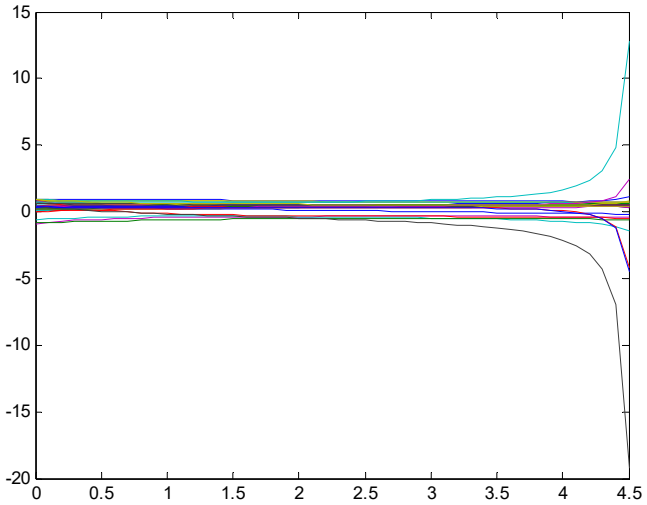

Nodes converge to consensus heading

# Trust-Based Control: Swarms/Formations

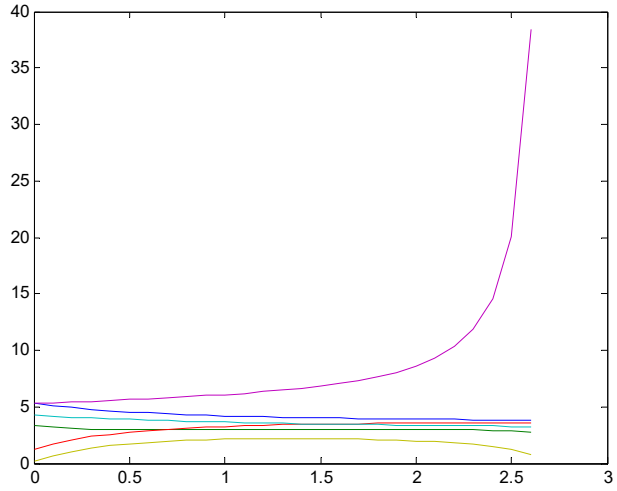
## Malicious Node

$$\dot{\theta}_i = \sum_{j \in N_i^c} \xi_{ij} a_{ij} (\theta_j - \theta_i)$$



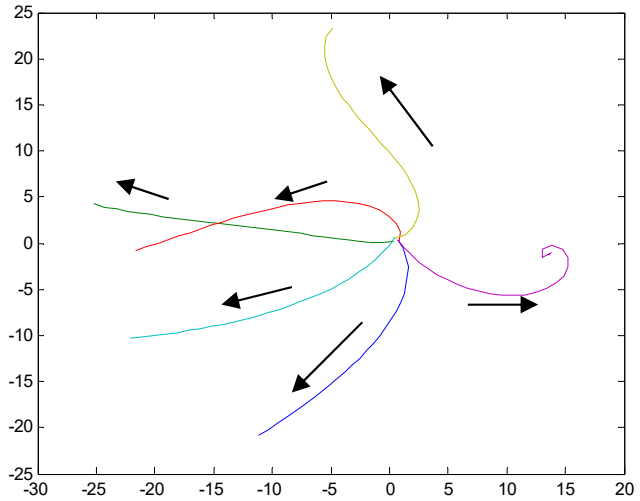




Divergence of trust




Divergence of headings

Node 5 injects
negative trust values



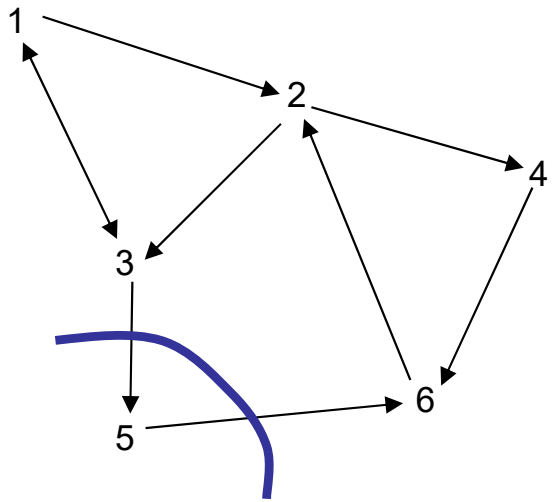

Causes Unstable Formation

Internal attack
Malicious node puts out bad trust values
i.e. false *information*
c.f. virus propagation

# Trust-Based Control: Swarms/Formations
# CUT OUT Malicious Node

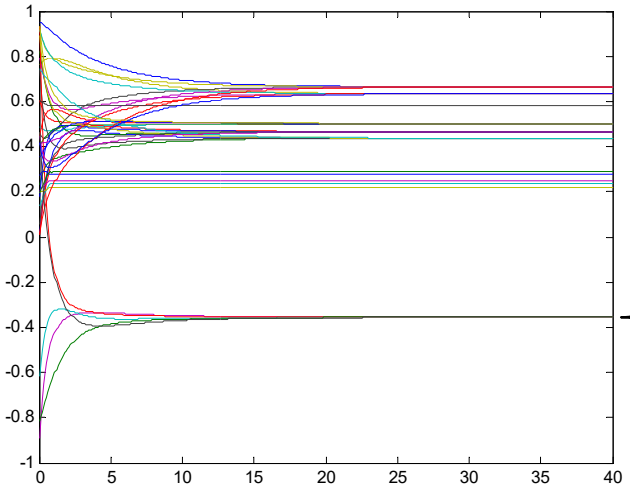
$$\dot{\theta}_i = \sum_{j \in N_i^c} \xi_{ij} a_{ij} (\theta_j - \theta_i)$$ heading angle

Work by Sajal Das



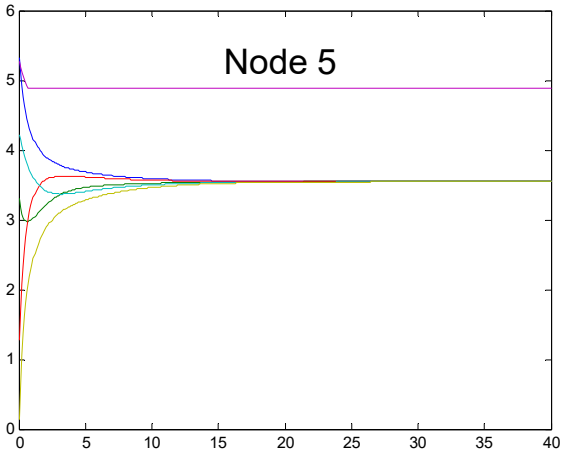

Node 5 injects
negative trust values

If node 3 distrusts node 5,
Cut out node 5

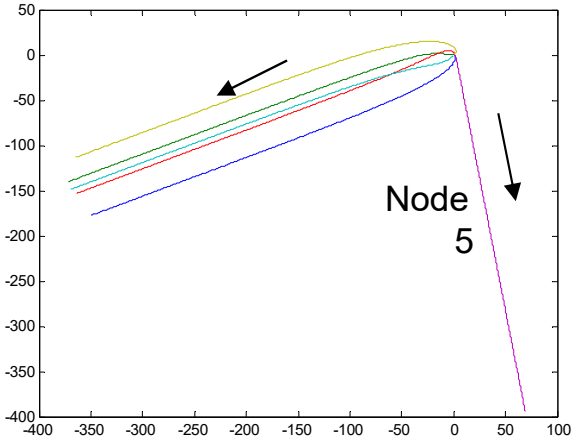


Other nodes agree that
node 5 has negative trust

Convergence of trust




Convergence of headings

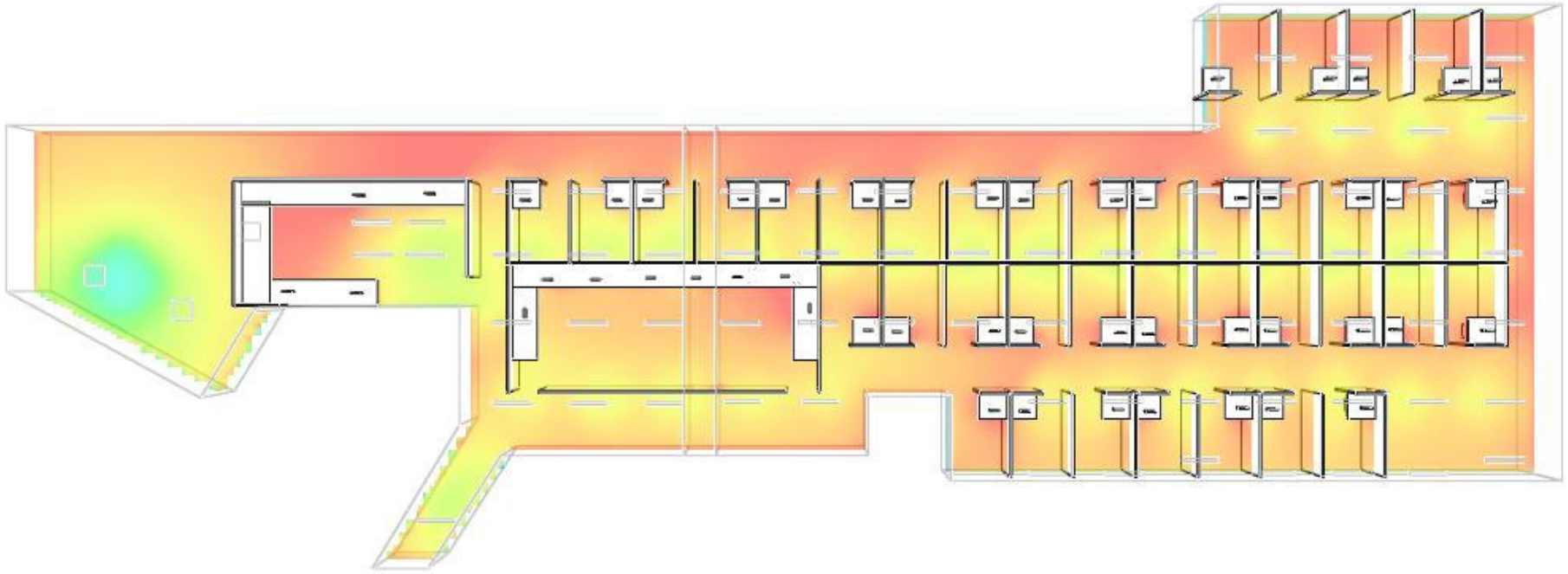



Restabilizes Formation



# Balancing HVAC Ventilation Systems

## Work with SIMTech – Singapore Inst. Manufacturing Technology

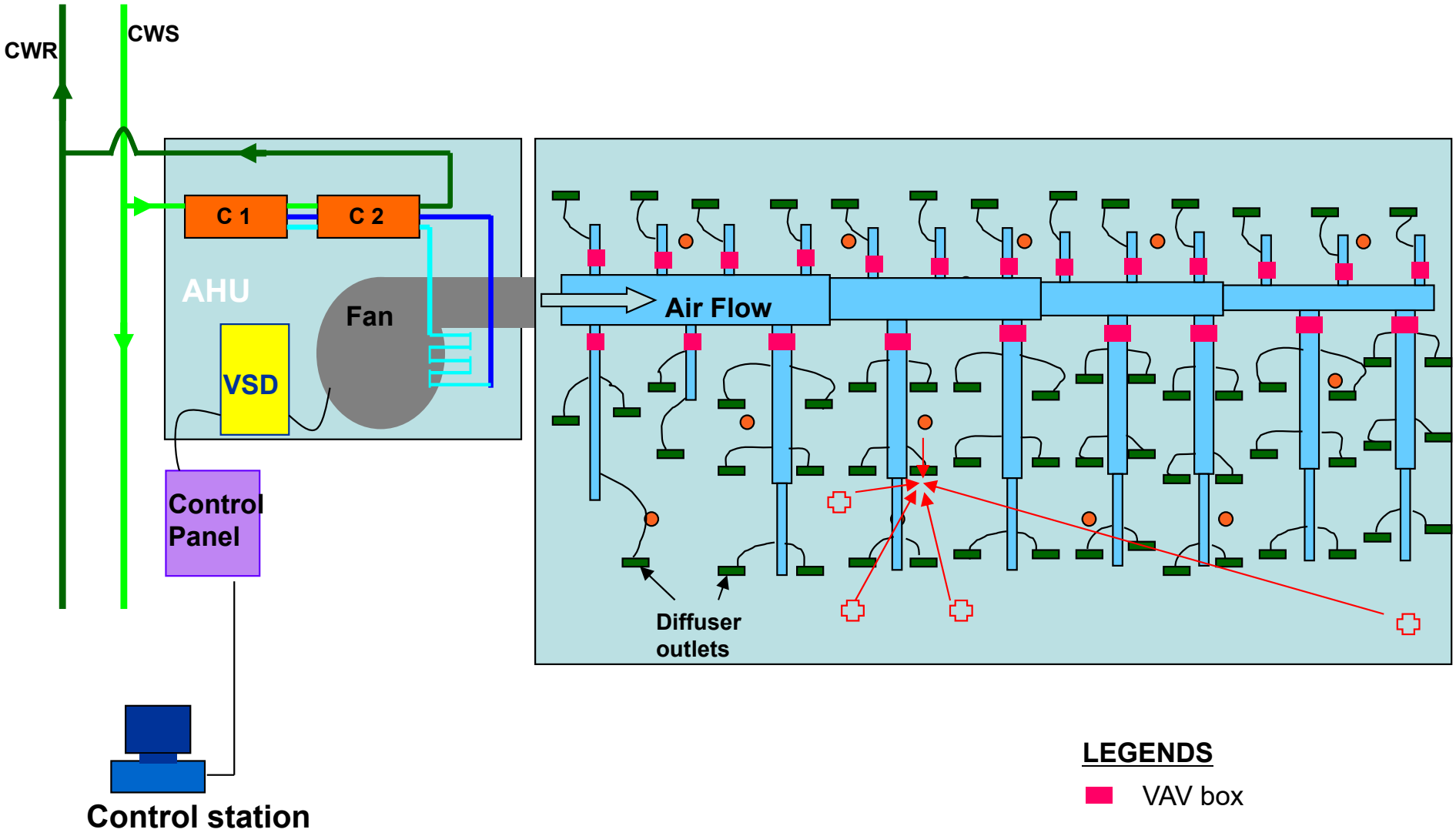


SIMTech 5th floor temperature distribution



SIMTech

# Automated VAV control system



# Adjust Dampers for desired Temperature distribution

Temperature dynamics

$$x_i(k+1) = x_i(k) + f_i(x) + u_i(k)$$

Unknown $f_i(x)$

Control damper position based on local voting protocol

$$u_i(k) = \frac{1}{n_i + 1}\gamma_i(k)\sum_{j \in N_i} a_{ij}(x_j(k) - x_i(k))$$

$$\gamma_i(k) = 1, \tfrac{1}{2}, \tfrac{1}{4}, \ldots$$

Under certain conditions this converges to steady-state desired temp. distribution

Open Research Topic - HVAC Flow and Pressure control

# Herd and Panic Behavior During Emergency Building Egress
## Work with Bari Polytechnic University, Italy (David Naso) – comes April 24

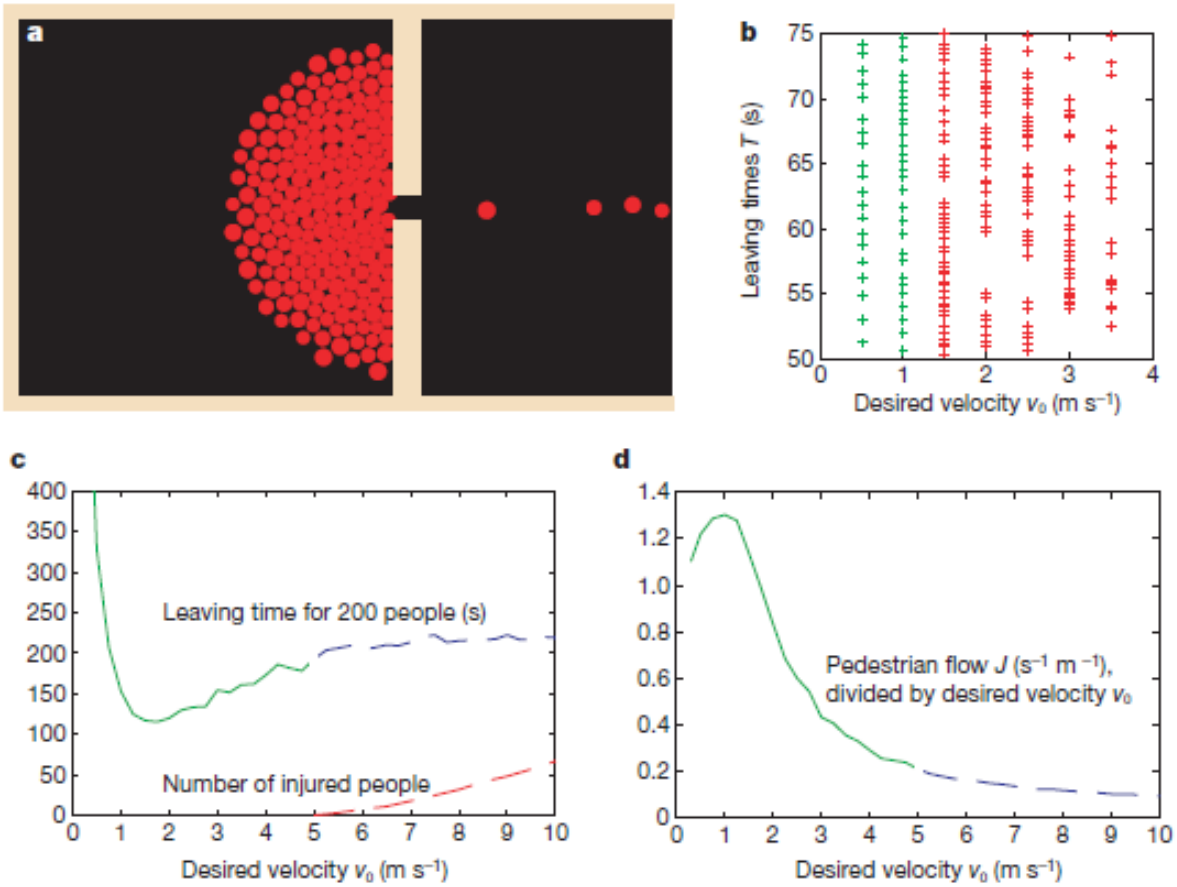
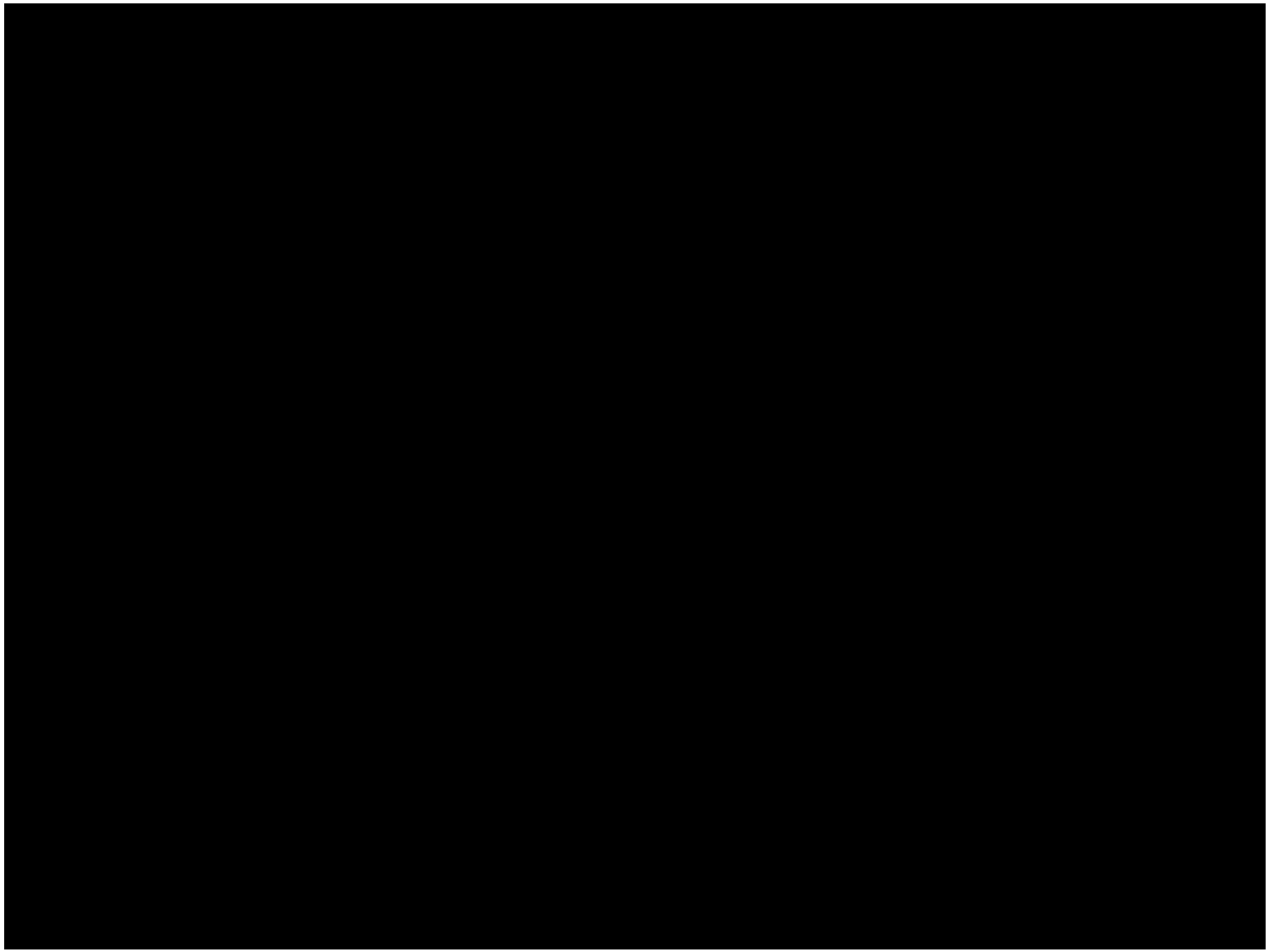



**Figure 1** Simulation of pedestrians moving with identical desired velocity $v_i^0 = v_0$ towards the 1-m-wide exit of a room of size 15 m × 15 m. **a**, Snapshot of the simulation. Dynamic simulations are available at http://angel.elte.hu/~panic/. **b**, Leaving times of pedestrians for various desired velocities $v_0$. Irregular outflow due to clogging is observed for high desired velocities ($v_0 \geq 1.5\,\mathrm{m\,s^{-1}}$, red plus signs). **c**, Under conditions of normal walking, the time for 200 pedestrians to leave the room decreases with growing $v_0$. Desired velocities higher than $1.5\,\mathrm{m\,s^{-1}}$ reduce the efficiency of leaving, which becomes particularly clear when the outflow $J$ is divided by the desired velocity (**d**). This is due to pushing, which causes additional friction effects. Moreover, above a desired velocity of about $v_0 = 5\,\mathrm{m\,s^{-1}}$ (corresponding to dashed lines in **c** and **d**) people are injured and become non-moving obstacles for others, if the sum of the magnitudes of the radial forces acting on them divided by their circumference exceeds a pressure of $1{,}600\,\mathrm{N\,m^{-1}}$ (ref. 5).

Owing to the above 'faster-is-slower effect', panics can be triggered by pedestrian counterflows[2], which cause delays to the crowd intending to leave. This makes the stopped pedestrians impatient and pushy which may be described by increasing the desired velocity according to $v_i^0(t) = [1 - p_i(t)]v_i^0(0) + p_i(t)v_i^{\max}$, where $v_i^0(0)$ is the initial, and $v_i^{\max}$ the maximum desired velocity. The time-dependent parameter $p_i(t) = 1 - \bar{v}_i(t)/v_i^0$, where $\bar{v}_i(t)$ denotes the average speed in the desired direction of motion, is a measure of impatience. Altogether, long waiting times increase the desired velocity, which can produce inefficient outflow. This further increases the waiting times, and so on, so that this tragic feedback can eventually trigger panics. It is therefore imperative to have sufficiently wide exits and to prevent counterflows when big crowds want to leave.

Helbring, Farkas, Vicsek, Nature 2000

# Crowd Panic Behavior



# Modeling Crowd Behavior in Stress Situations

Helbring, Farkas, Vicsek, Nature 2000

Consensus term

Interaction pot. field

Wall pot. field

$$m_i \frac{\mathrm{d}\mathbf{v}_i}{\mathrm{d}t} = m_i \frac{v_i^0(t)\mathbf{e}_i^0(t) - \mathbf{v}_i(t)}{\tau_i} + \sum_{j(\neq i)} \mathbf{f}_{ij} + \sum_W \mathbf{f}_{iW}$$

desired speed $v_i^0$ in a certain direction $\mathbf{e}_i^0$,

Repulsive force

Radial compression term

Tangential friction term

$$\mathbf{f}_{ij} = \left\{A_i \exp[(r_{ij} - d_{ij})/B_i] + kg(r_{ij} - d_{ij})\right\}\mathbf{n}_{ij} + \kappa g(r_{ij} - d_{ij})\Delta v_{ji}^t \mathbf{t}_{ij} \quad (2)$$

where the function $g(x)$ is zero if the pedestrians do not touch each other ($d_{ij} > r_{ij}$), and is otherwise equal to the argument $x$.

# Controlled Consensus: Cooperative Tracker

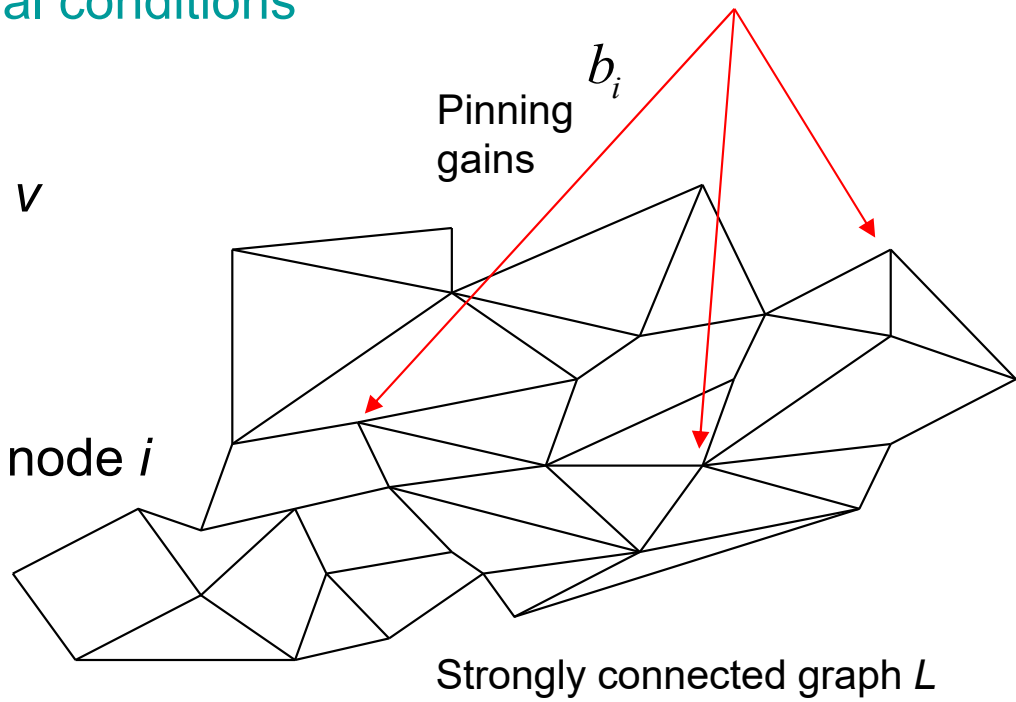
Ron Chen

## Get rid of dependence on initial conditions

Node state $\dot{x}_i = u_i$

Local voting protocol with LEADER node *v*

$$u_i = \sum_{j \in N_i} a_{ij}(x_j - x_i) \boxed{+ b_i(v - x_i)}$$




$b_i \neq 0$ If Leader v is in the neighborhood of node *i*

$$u_i = -\left( b_i + \sum_{j \in \bar{N}_i} a_{ij} \right) x_i + \sum_{j \in N_i} a_{ij} x_j \quad + b_i v$$

Leaderl node is in some neighborhoods $\bar{N}_i$

Pinning Control

$$\dot{x} = -(L + B)x + B\underline{1}v \qquad B = diag\{b_i\}$$

Theorem. Let at least one $b_i \neq 0$. Then L+B is nonsingular with all e-vals positive and -(L+B) is asymptotically stable

So initial conditions of nodes in graph A go away.
Consensus value depends only on v

In fact, v is now the only spanning node

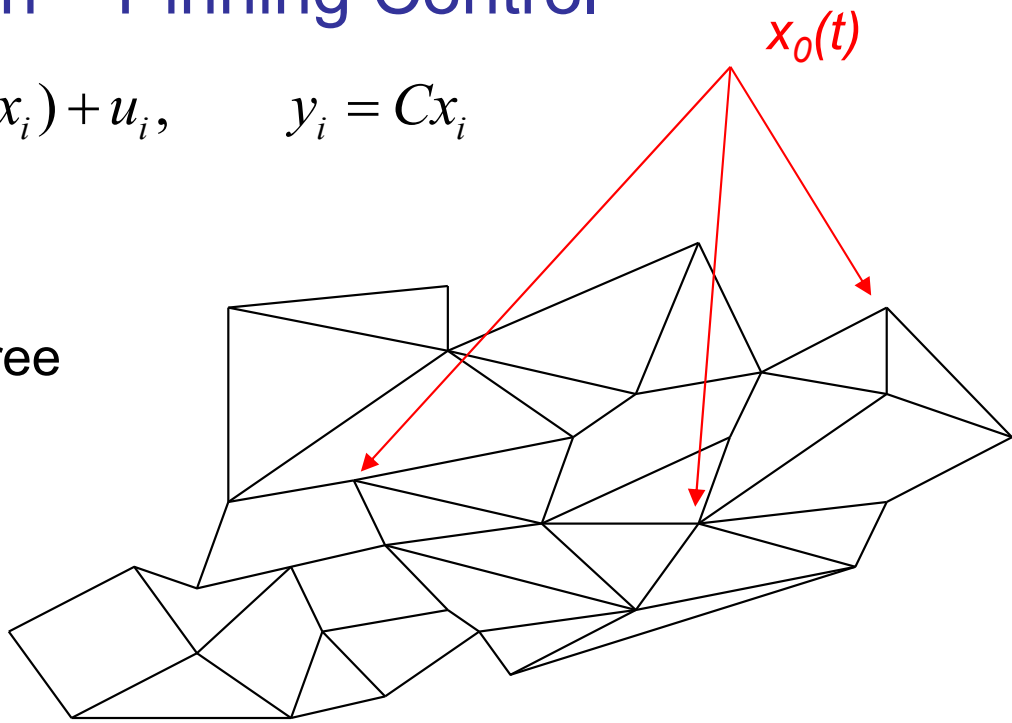
# Synchronization : Ron Chen – Pinning Control

Connected undirected graphs $\dot{x}_i = f(x_i) + u_i, \qquad y_i = Cx_i$

Leader node dynamics $\dot{x}_0 = f(x_0)$

$d_i = \sum_{j \neq i} a_{ij} = \sum_{j \neq i} a_{ji}$ In-degree = out-degree

$a_{ii} = -d_i$ Diffusivity condition




Pinning Control – inputs to some nodes

$$\dot{x}_i = f(x_i) + c\sum_j a_{ij} C(x_j - x_i) + cb_i C(x_0 - x_i)$$

$b_i = 1$ if node *i* is pinned

$$\dot{x} = \left(f(x) - c(L+B)\otimes Cx\right) \quad + \quad cB \otimes C\underline{1}x_0$$

$B = diag(b_i)$

$L + B = D + B - A$ has e-vals $\lambda_i$

Results –
Node motions synchronize if $\left(\frac{\partial f(x)}{\partial x} - c\lambda_i C\right)$ is stable

Pin to the biggest node = highest degree node= highest social standing- c.f. Baras

Must have control gain c big enough

# Synchronization

Spong and Chopra

$$\dot{x}_i = f_i(x_i) + g(x_i)u_i$$
$$y_i = h(x_i)$$

passive

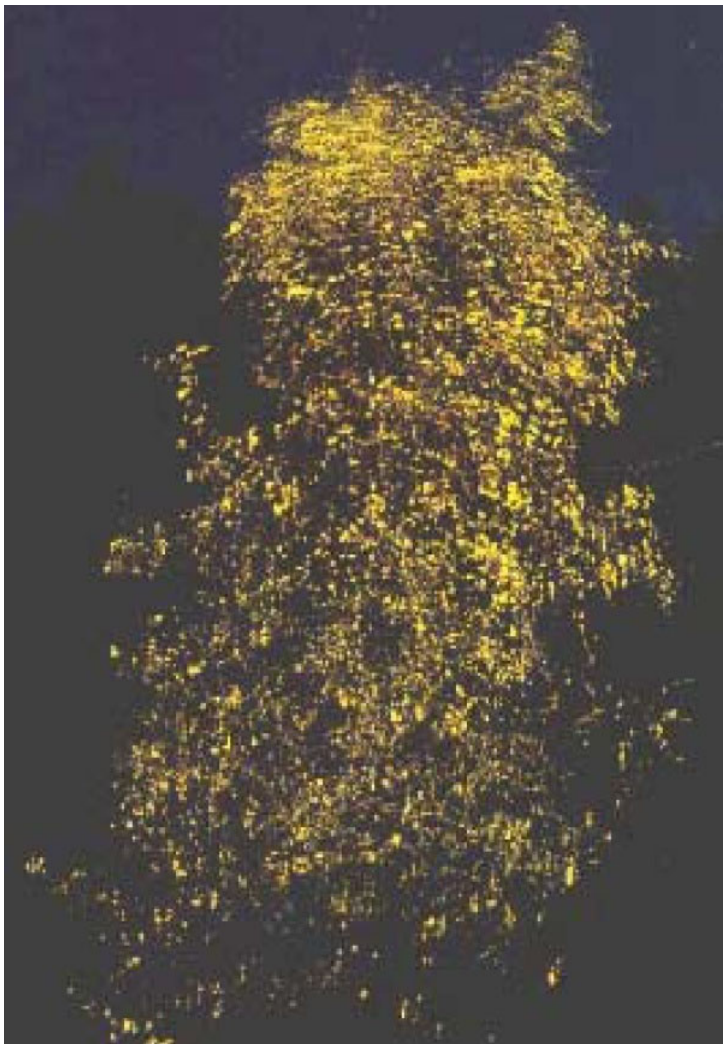
$$V_i(x_i) - V_i(x_i(0)) \le \int_0^t u_i^T(s) y_i(s)\, ds$$

Storage function

Synchronize if $y_i(t) \to y_j(t), \quad all\ i, j$

Local voting protocol with OUTPUT FEEDBACK

$$u_i = \sum_{j \in N_i} K(y_j - y_i)$$

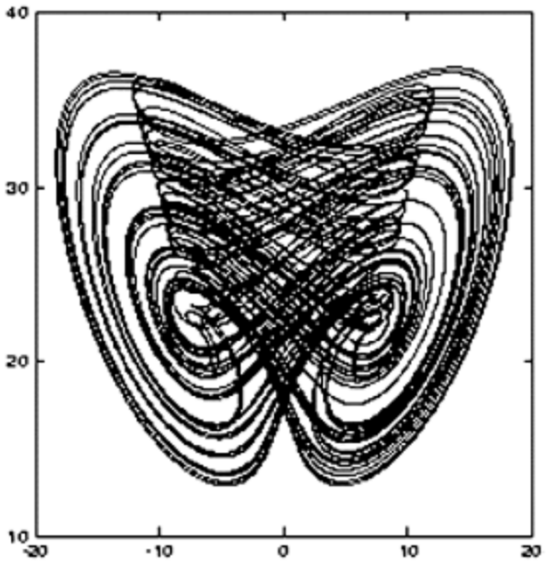


Fireflies synchronize

Result -
Let the communication graph be balanced. Then the agents synchronize.

Circadian rhythm

# Synchronization of Chaotic node dynamics – Ron Chen



Chen's attractor
node dynamics

Pinning control of largest node
(for increasing coupling strengths)

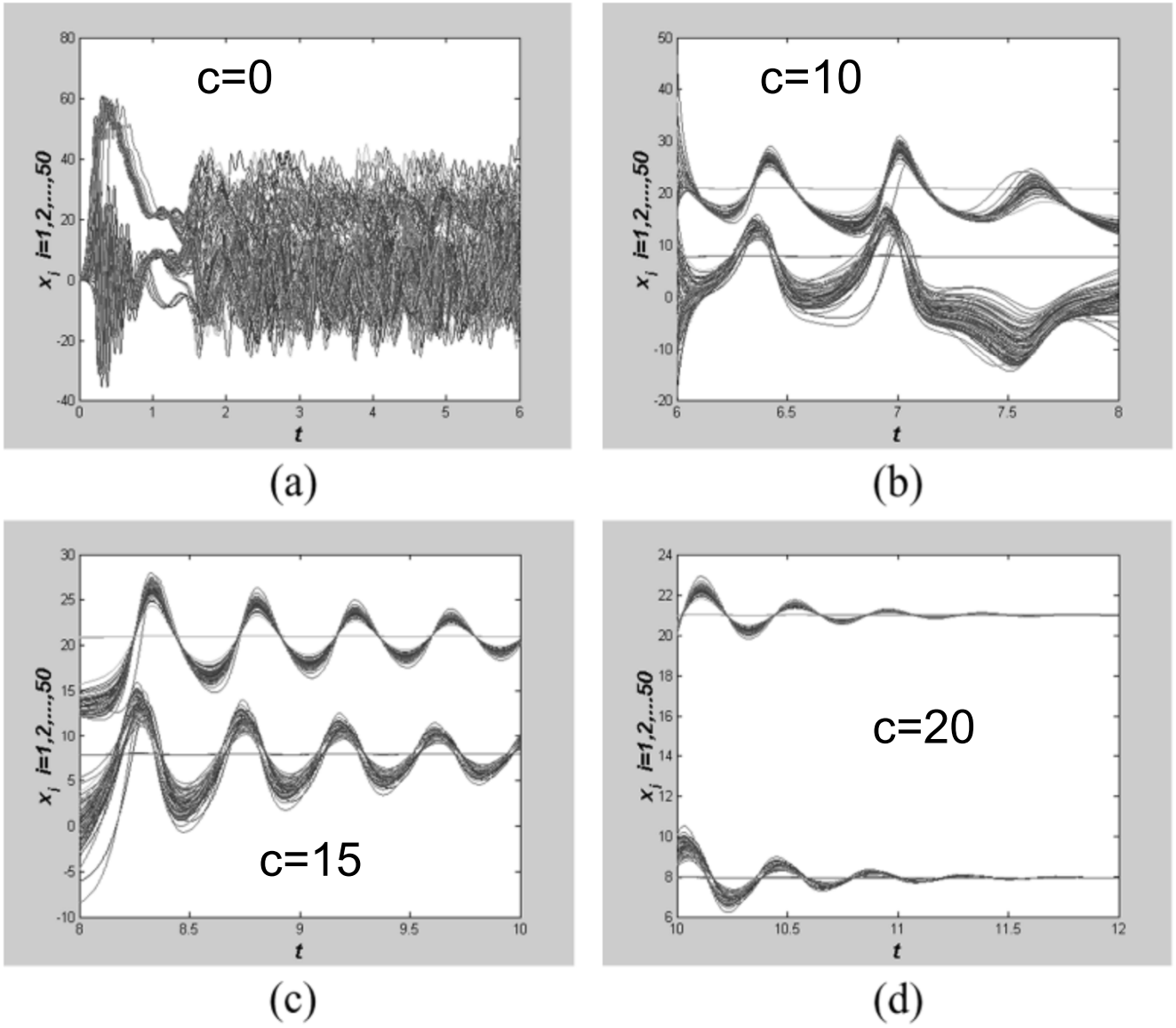



Fig. 8. Specifically pinning the biggest node of 19 degrees in a 50-node network generated by the B–A scale-free model: (a)–(d) are stabilizing phases with different coupling strengths. (a) $c = 0, d = 0$. (b) $c = 10, d = 1000$. (c) $c = 15, d = 1000$. (d) $c = 20, d = 1000$.

# Kung Tz 500 BC

## Confucius

孔子

Man's relations to
- Family
- Friends
- Society
- Nation
- Emperor
- Ancestors

Tian xia da tong
Harmony under heaven

Archery
Chariot driving

Music
Rites and Rituals

Poetry
Mathematics






124 BC - Han Imperial University in Chang-an

Our revels now are ended. These our actors,
As I foretold you, were all spirits, and
Are melted into air, into thin air.

The cloud-capped towers, the gorgeous palaces,
The solemn temples, the great globe itself,
Yea, all which it inherit, shall dissolve,
And, like this insubstantial pageant faded,
Leave not a rack behind.

We are such stuff as dreams are made on,
and our little life is rounded with a sleep.

Prospero, in The Tempest, act 4, sc. 1, l. 152-6, Shakespeare

