EE 5330 Homeworks

Updated: Wednesday, November 16, 2022

DO NOT DO HOMEWORK UNTIL IT IS ASSIGNED.
THE ASSIGNMENTS MAY CHANGE UNTIL ANNOUNCED.

- For full credit, show all work.
- Some problems require hand calculations. In those cases, do not use MATLAB except to check your answers.

It is OK to talk about the homework beforehand.

BUT, once you start writing the answers, MAKE SURE YOU WORK ALONE.

The purpose of the Homework is to evaluate you individually, not to evaluate a team.

Cheating on the homework will be severely punished.

The next page must be signed and turned in at the front of ALL homeworks submitted in this course.

Homework Pledge of Honor

On all homeworks in this class - YOU MUST WORK ALONE.

Any cheating or collusion will be severely punished.

It is very easy to compare your software code and determine if you worked together
It does not matter if you change the variable names.

Please sign this form and include it as the first page of all of your submitted homeworks.

.....

Typed Name: _____

Pledge of honor:

"On my honor I have neither given nor received aid on this homework."

Signature: _____

EE 5330 Homework 1

State Variable Systems, Computer Simulation

1. Simulate the van der Pol oscillator $y''+\alpha(y^2-1)y'+y=0$ using MATLAB for various ICs. Plot for 100 seconds. In each case below, first use ICs of y(0)=0, y'(0)= 0.5. Then use ICs of y(0)=4, y'(0)= 4.
   Plot y(t) vs. time *t* and also the phase plane plot y'(t) vs. y(t).
   a. For α= 0.05.
   b. For α= 0.9.

2. Do MATLAB simulation of the Lorenz Attractor chaotic system. Run for 150 sec. with all initial states equal to 0.5. Plot states versus time, and also make 3-D plot of x_1, x_2, x_3 using PLOT3(x1,x2,x3).

$$\dot{x}_1 = -\sigma(x_1 - x_2)$$
$$\dot{x}_2 = rx_1 - x_2 - x_1x_3$$
$$\dot{x}_3 = -bx_3 + x_1x_2$$

use σ= 10, r= 28, b= 8/3.

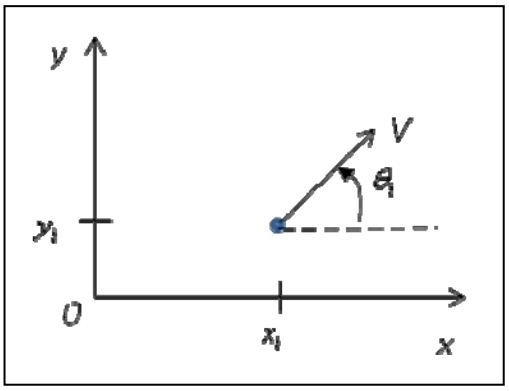
3. Consider the Voltera predator-prey system

$$\dot{x}_1 = -x_1 + x_1x_2$$
$$\dot{x}_2 = x_2 - x_1x_2$$

.

Simulate the system using MATLAB for various initial conditions. Take ICs spaced in a uniform mesh in the box x1=[-2,2], x2=[-2,2]. Make one phase plane plot with all the trajectories on it. Plot phase plane on square [-5,5]x[-5,5].

Consensus for Formation Control



Let each node have the vehicle dynamics given by

$$\dot{x}_i = V\cos\theta_i$$

$$\dot{y}_i = V\sin\theta_i$$

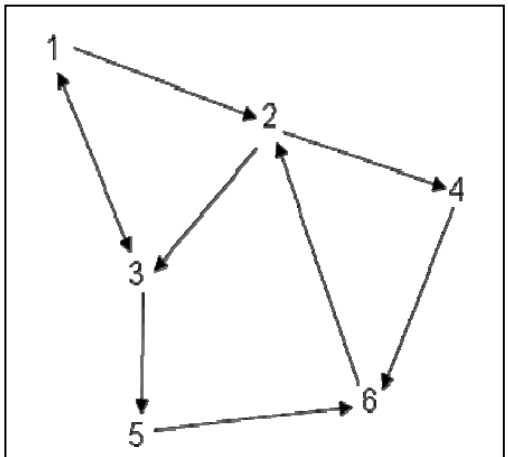
with $(x_i(t), y_i(t))$ the position and $\theta_i(t)$ the heading. This corresponds to motion in the (x,y) plane with velocity V as shown. All nodes have the same velocity V= 1 m/sec.

Take a flock of 6 nodes with the strongly connected communication graph structure shown. Run the continuous-time local voting protocol on the headings of the nodes as

$$\dot{\theta}_i = \sum_{j\in N_i} a_{ij}(\theta_j - \theta_i)$$

Take all the edge weights equal to 0.5. Set the initial headings random between $0-2\pi$ rads.

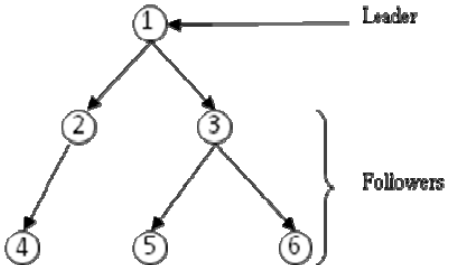
Now, you have 3 states running at each node: $\theta_i(t)$, $(x_i(t), y_i(t))$. Plot the six headings vs time. They should reach the weighted average consensus. Plot the six positions $(x_i(t), y_i(t))$ of the nodes in the plane vs time. They should eventually all go off in the same direction.



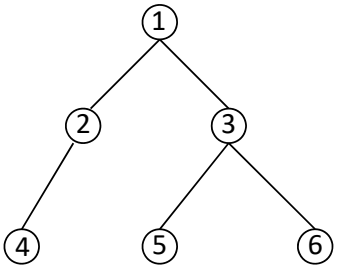
Graph for formation control.

EE 5330 Homework 3
Graph Laplacian Eigenvalues and Eigenvectors

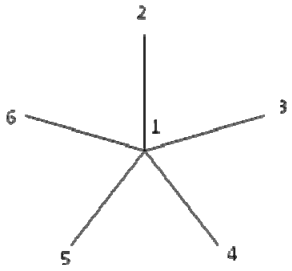
For the following graphs, take all edge weights equal to 1. (a) Write the adjacency matrix A and the graph Laplacian L. (b) Find the eigenvalues of L and plot in the complex s-plane. (c) Compare the Fiedler e-vals λ_2. (d) Find the left eigenvector w_1 of L for $\lambda_1 = 0$.



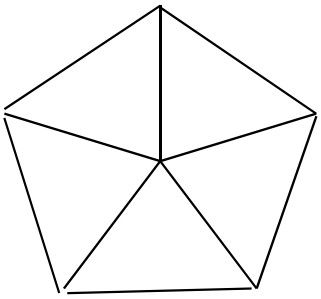
1. Directed Tree graph. (Formation graph)

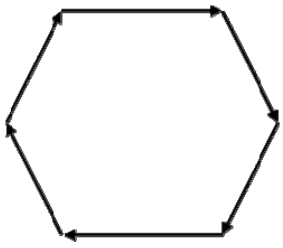


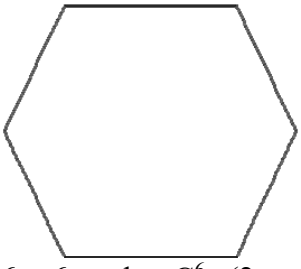
2. Undirected Tree graph.

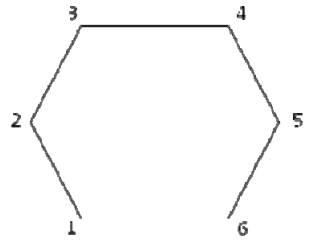


3. Star $K_{1,5}$


4. Star with extra edges


5. Directed 6-cycle (6-periodic graph)


6. 6-cycle C^6 (2-regular graph).



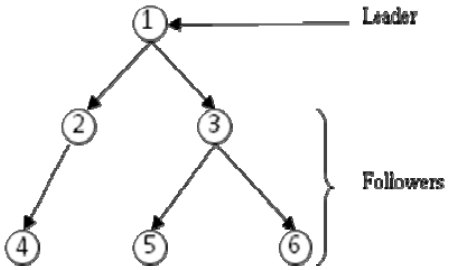
7. Path P^5.

EE 5330 Homework 4
Consensus for Different Graph Eigenvalues

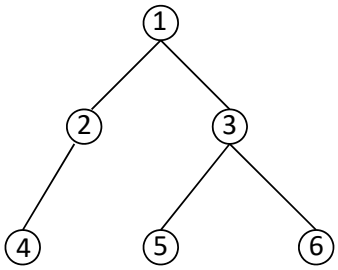
Simulate the continuous-time consensus protocol

$$\dot{x}_i = u_i = \sum_{j \in N_i} a_{ij}(x_j - x_i).$$

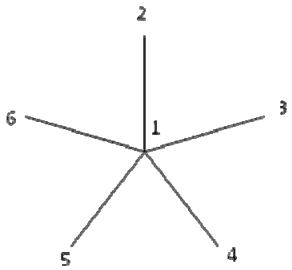
for all the graphs shown below. Take all edge weights equal to 1. For each case, plot all six of the states versus time on the same plot. Take random initial conditions.



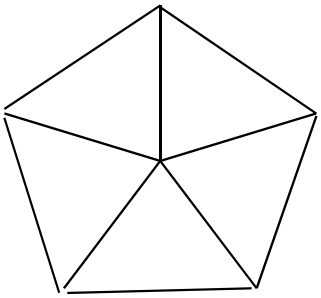
1. Directed Tree graph. (Formation graph)



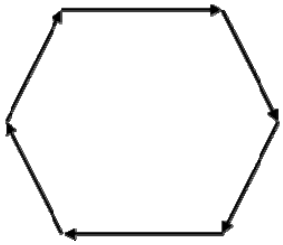
2. Undirected Tree graph.



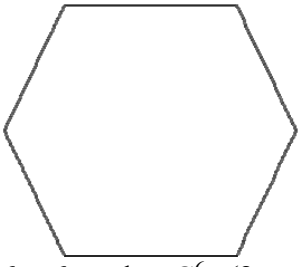
3. Star $K_{1,5}$



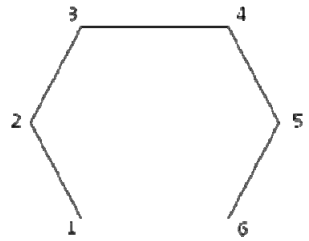
4. Star with extra edges



5. Directed 6-cycle (6-periodic graph)



6. 6-cycle C^6 (2-regular graph).



7. Path P^5.

EE 5330 Exam 1 – Takehome Exam
Kuramoto Oscillators

Make sure you turn in the Pledge of Honor with this exam.

N coupled Kuramoto Oscillators have dynamics

$$\dot{\theta}_i = \omega_i + \frac{K}{N}\sum_{j\in N_i} a_{ij}(\theta_j - \theta_i)$$

with θ_i the phases, ω_i the natural oscillation frequency of oscillator *i,* and *K* the coupling gain.

1. Simulation

Take 6 oscillators coupled on the graph shown. Take all edge weights equal to $1/2$. Take natural frequencies of $\omega_1 = 1, \omega_2 = 1.1, \omega_3 = 0.9, \omega_4 = 0.85, \omega_5 = 2, \omega_6 = 2.1$.

You will do several simulations. Take random initial phases $\theta_i(0)$.

a. Set K=0 and

i. plot phase θ_i(t) vs time t, all on one graph.

ii. plot frequency $\dot{\theta}_i$(t) vs time t, all on one graph.

iii. make the phase plane plot $\dot{\theta}_i(t)$ vs $\theta_i(t)$ all on one graph.

You should see no synchronization.

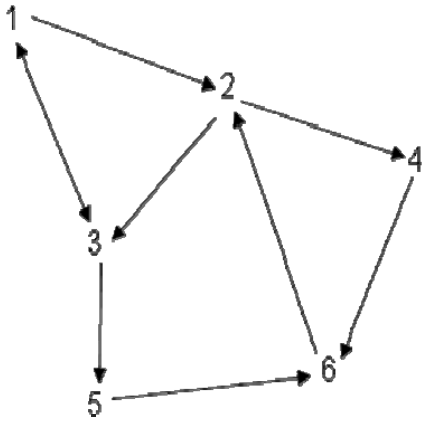
Repeat for

b. K=2

c. K= 5

d. K= 50

As K increases synchronization should improve. At K=50 all agents should synchronize



Graph for formation control.

2. Discussion

Explain thoroughly the simulation results.

You need to do a good job to get full credit. Work alone on this.

Hints:

Find the average of the natural frequencies ω_i . Analyze the behavior in terms of how far the agents are from average frequency.

Discuss the effects of initial phases $\theta_i(\mathrm{t})$

EE 5330 Homework 5

Discrete-time Consensus

Simulate the DT consensus protocol using the normalized form protocol

$$x_i(k+1) = x_i(k) + \frac{1}{d_i+1}\sum_{j\in N_i} a_{ij}(x_j(k) - x_i(k)) = \frac{1}{d_i+1}\left(x_i(k) + \sum_{j\in N_i} a_{ij}x_j(k)\right)$$

on graphs 1 and 5 from Homework 4. Take all edge weights equal to 1//2. For each case, plot all the states versus time. Start with random initial condition states between -1, +1.

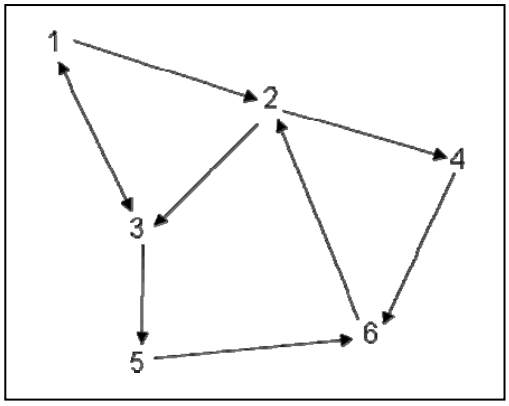
EE 5330 Exam 2
Time-varying Graph Topologies

1. Discrete-Time Consensus with Varying Graph Topology

We want to simulate DT consensus for the normalized protocol systems

$$x_i(k+1) = x_i(k) + \frac{1}{d_i+1}\sum_{j\in N_i} a_{ij}(x_j(k) - x_i(k))$$

$$= \frac{1}{d_i+1}\left(x_i(k) + \sum_{j\in N_i} a_{ij}x_j(k)\right)$$



The global form of this is

$$x(k+1) = x(k) - (I+D)^{-1}Lx(k)$$

$$= (I+D)^{-1}(I+A)x(k) \equiv Fx(k)$$

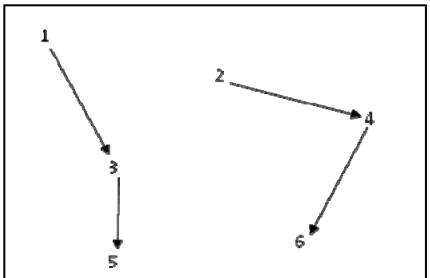
a. Simulate this algorithm for the strongly connected digraph shown above. Start with random initial conditions between -1, +1. Take edge weights of 1.

b. Shown below are three disconnected graphs whose union is the original strongly connected graph.

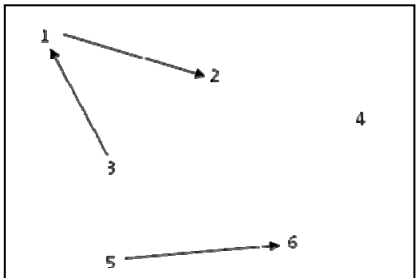
Simulate the DT protocol for the case of time-varying protocols

$$x(k+1) = F(k)x(k) = (I+D(k))^{-1}(I+A(k))x(k)$$

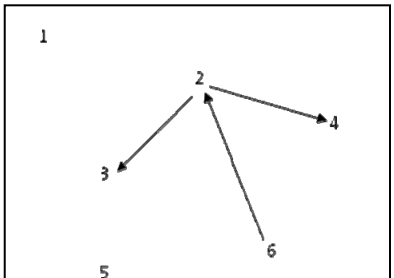
where *F(0)* corresponds to graph G1, *F(1)* to Graph G2, *F(2)* to G3, and then this cycle 123 123 is repeated. Use the same initial conditions as in part 1a. Plot the states vs. time. Verify that consensus is achieved. Are the consensus values the same as in Problem 1a?



Graph G1



Graph G2



Graph G3

c. Repeat for a different order of switching between the three graphs. Use the cycle 12222233333 12222233333 ad infinitum. Use the same initial conditions as in part 1a. Compare to part a. Is the consensus value the same?

2. Gossip Algorithms

You do not have to do this problem unless you want to
It is for extra credit

Use the same strongly connected graph as in problem 1. Simulate the gossip algorithm for average consensus. That is, at each step, select an edge at random. Update the states of the two nodes joined by the edge according to

$$x_i = x_i + \frac{x_j - x_i}{2}, \qquad x_j = x_j - \frac{x_j - x_i}{2}$$

Start with random initial conditions. Plot the states vs. time. Verify that consensus is reached. The consensus value should be the average of the initial states.