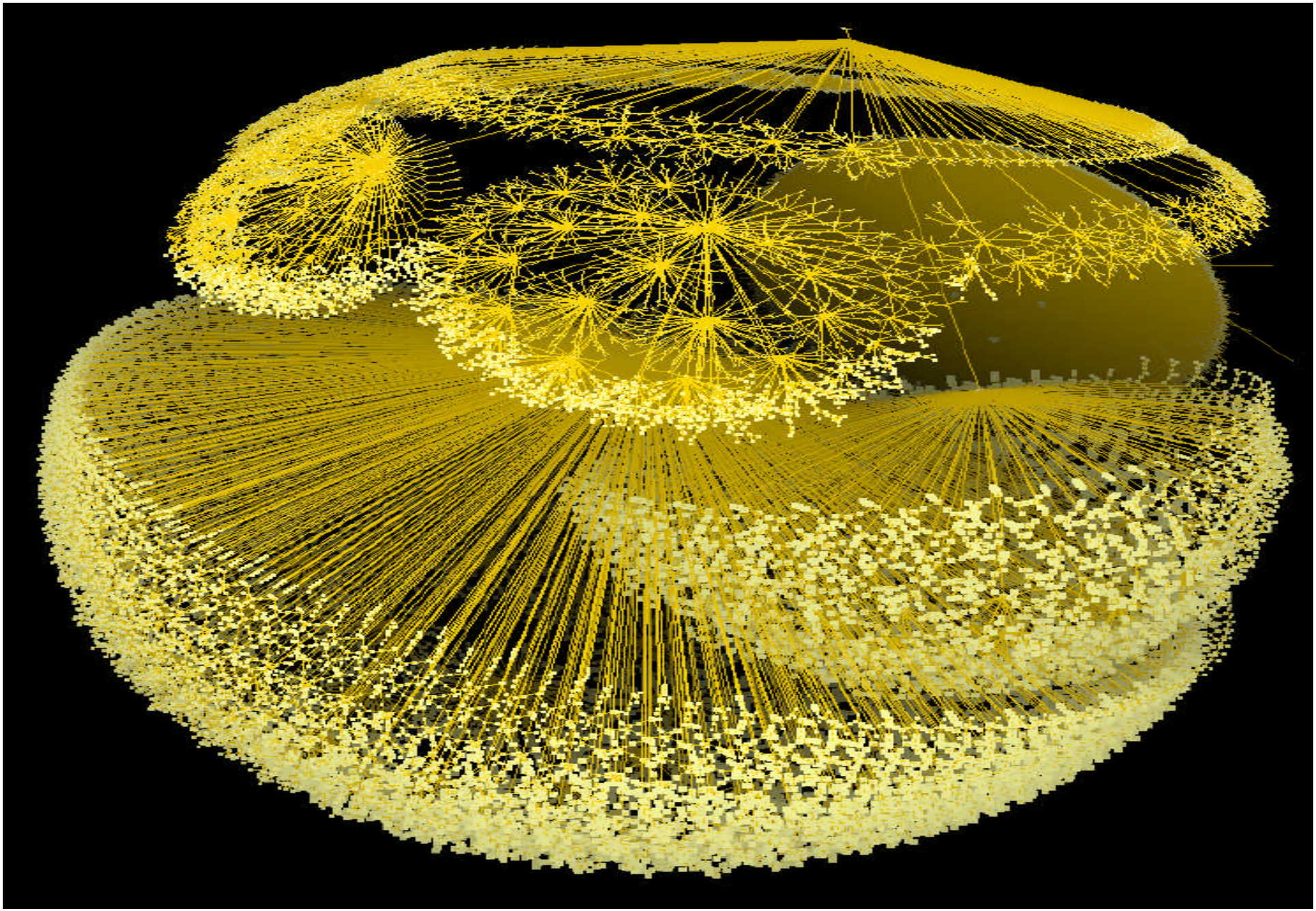






Distributed Cooperative Control of AC Micro-grids



F.L. Lewis, NAI

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Distributed Synchronization Control of AC Micro-grids

Work of Ali Bidram with Dr. A. Davoudi

Supported by :
China Qian Ren Program, NEU
China Education Ministry Project 111 (No.B08015)
US NSF - Paul Werbos



Talk available online at
http://www.UTA.edu/UTARI/acs

UTARI
UNIVERSITY OF TEXAS AT ARLINGTON
RESEARCH INSTITUTE

Invited by Lu Liu



F.L. Lewis, H. Zhang, A. Das, K. Hengster-Movric, *Cooperative Control of Multi-Agent Systems: Optimal Design and Adaptive Control*, Springer-Verlag, 2013



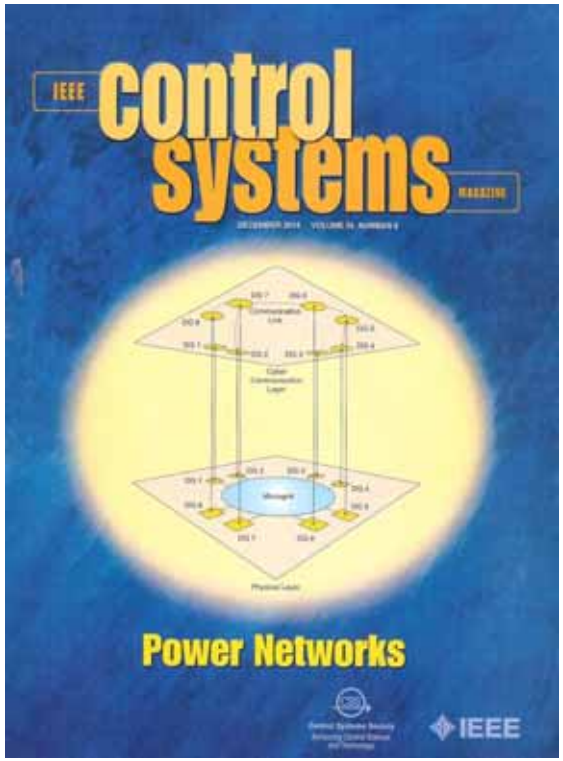
Key Point

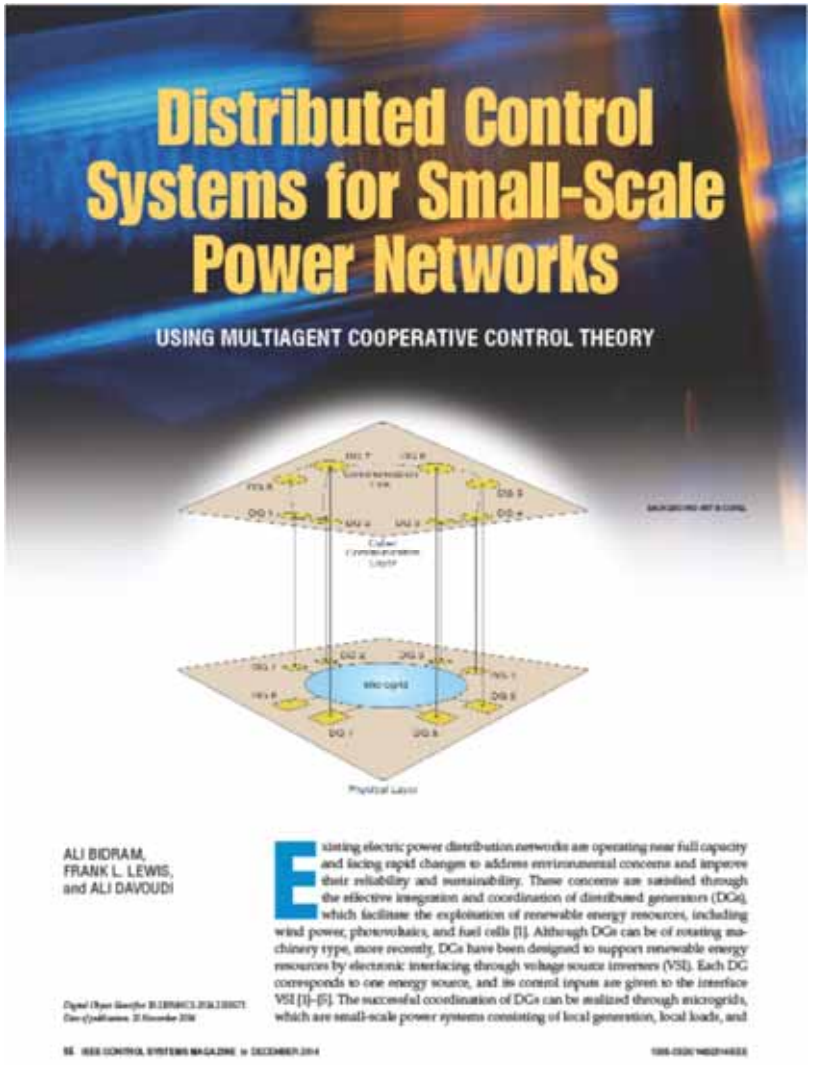
Lyapunov Functions and Performance Indices
Must depend on graph topology

H. Zhang, F.L. Lewis, and Z. Qu, "Lyapunov, Adaptive, and Optimal Design Techniques for Cooperative Systems on Directed Communication Graphs," IEEE Trans. Industrial Electronics, vol. 59, no. 7, pp. 3026-3041, July 2012.

Hongwei Zhang, F.L. Lewis, and Abhijit Das
"Optimal design for synchronization of cooperative systems: state feedback, observer and output feedback," IEEE Trans. Automatic Control, vol. 56, no. 8, pp. 1948-1952, August 2011.

A. Bidram, F.L. Lewis, and A. Davoudi, “Distributed Control Systems for Small-scale Power Networks Using Multi-agent Cooperative Control Theory,” IEEE Control Systems Magazine, pp. 56-77, December 2014 ***(Featured cover article).***





Outline

Cooperative Control

Electric Power Microgrids

Cooperative Control for Synchronization in Microgrids

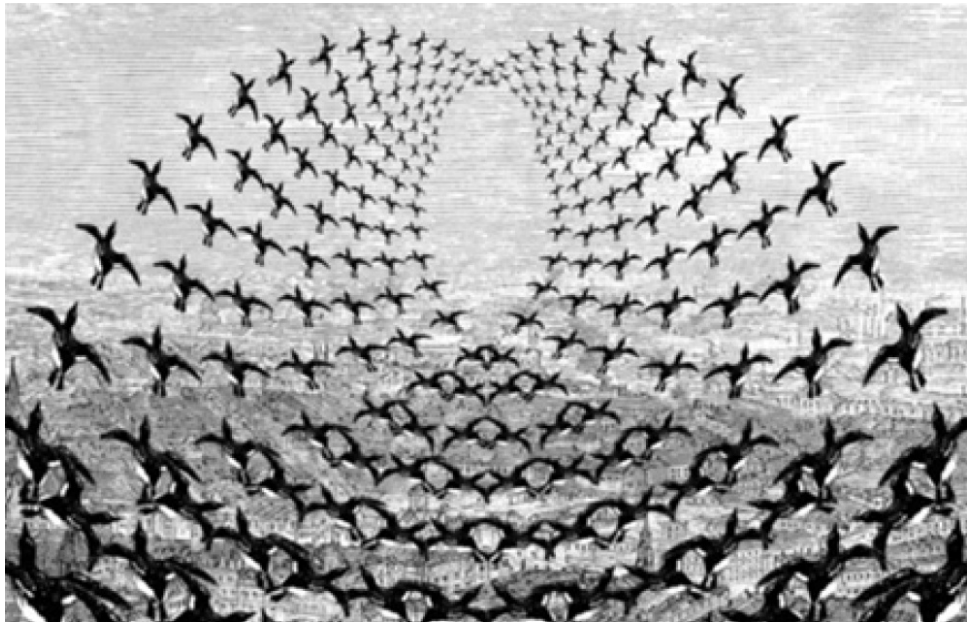


Synchronized Motion of biological groups



Fish school

Birds flock





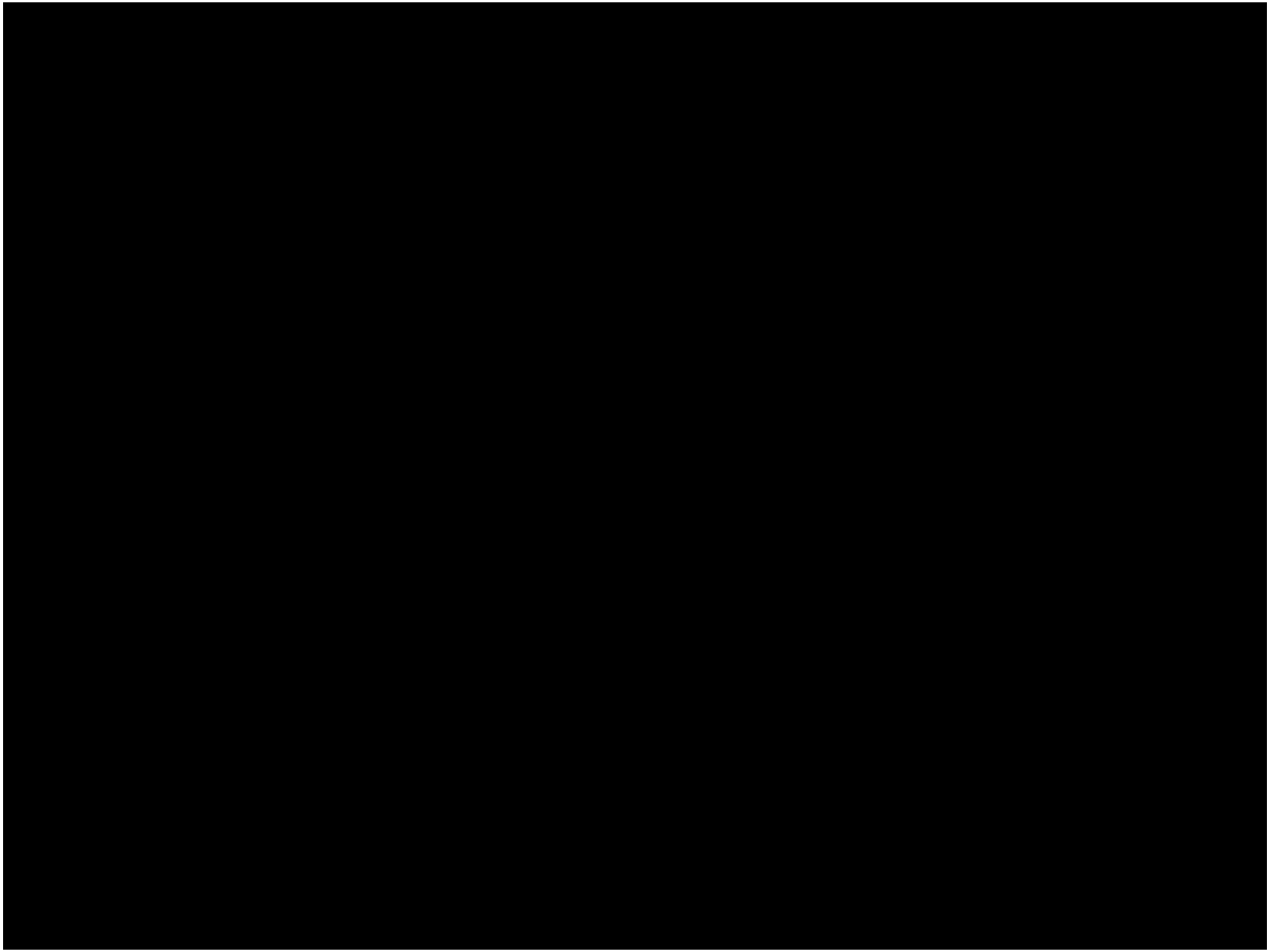
Locusts swarm

Fireflies synchronize

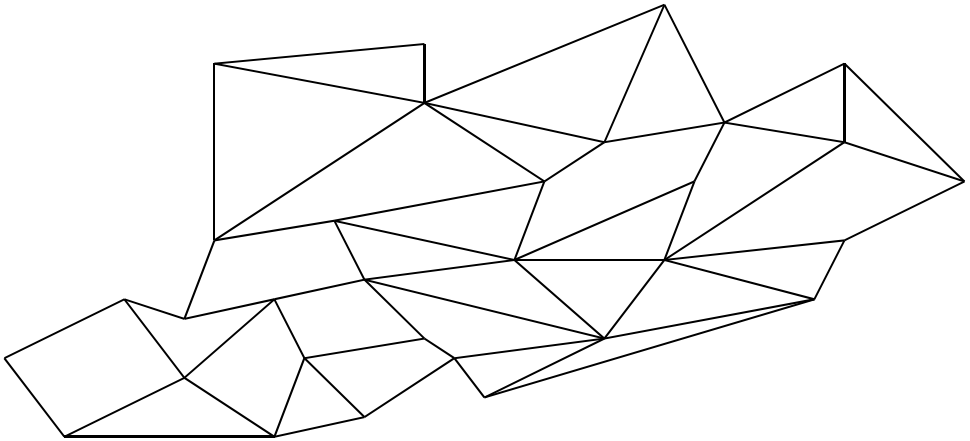


The Power of Synchronization

Coupled Oscillators
Diurnal Rhythm



Communication Graph

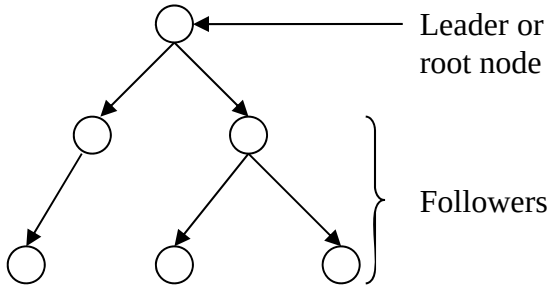


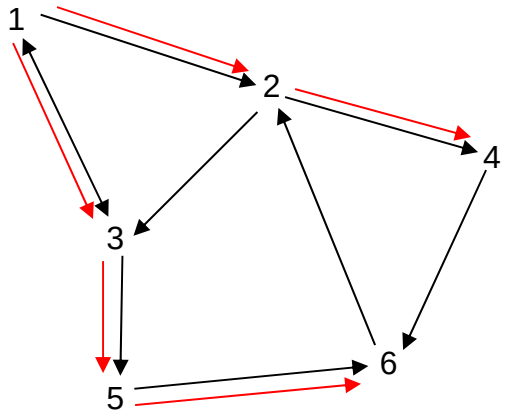
Strongly connected if for all nodes *i* and *j* there is a path from *i* to *j*.

Diameter= length of longest path between two nodes

Volume = sum of in-degrees $Vol = \sum_{i=1}^{N} d_i$

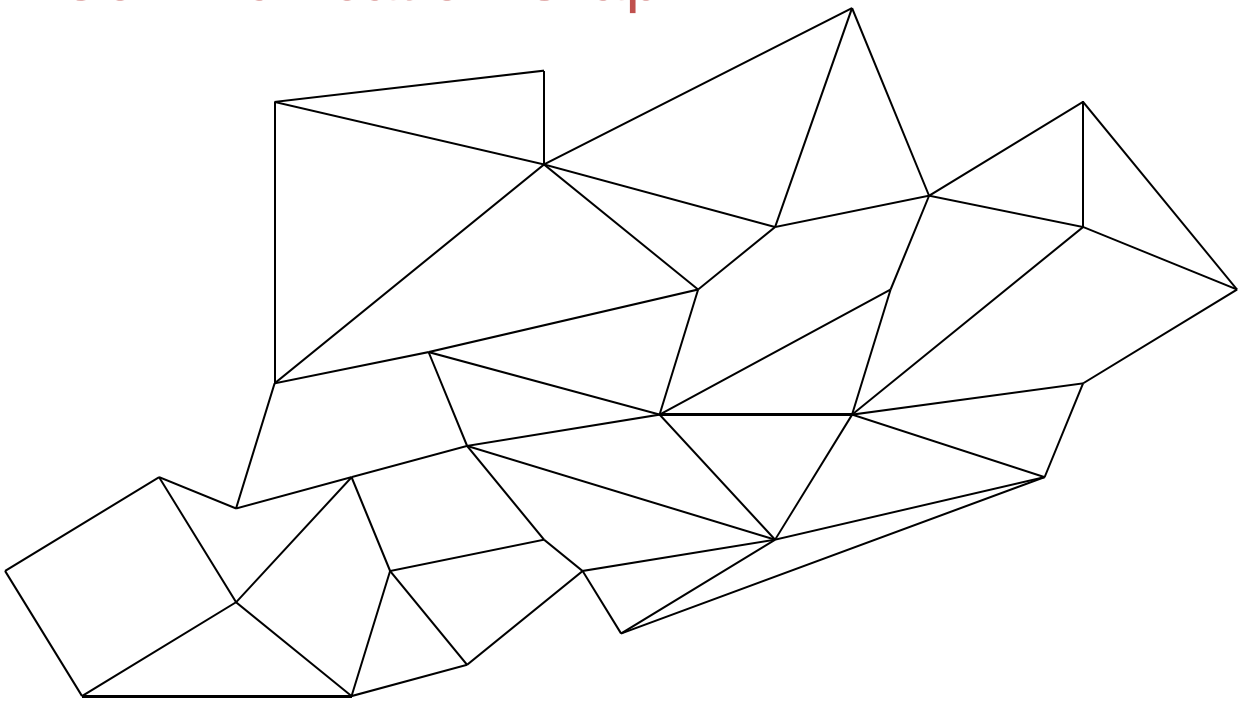
Tree- every node has in-degree=1





Spanning tree
Root node

Communication Graph



G=(V,E)

N nodes

State at node *i* is $x_i(t)$

Synchronization problem

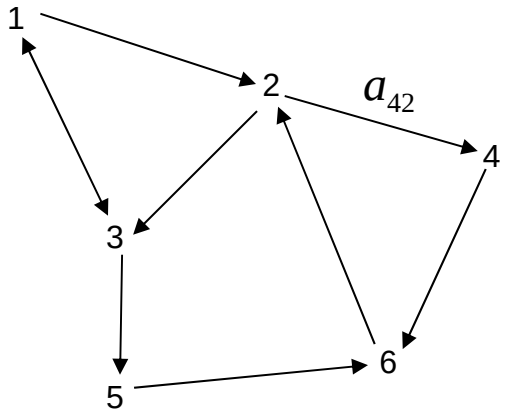
$$x_i(t) - x_j(t) \to 0$$

Communication Graph

Algebraic Graph Theory

(V,E)

N nodes



Adjacency matrix

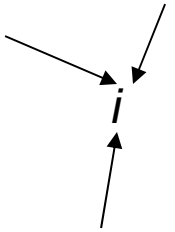
$$A = [a_{ij}]$$

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \end{bmatrix}$$

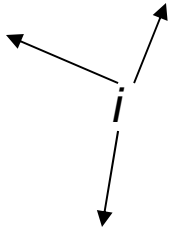
$$a_{ij} > 0 \; if \; (v_j, v_i) \in E$$

$$if \;\; j \in N_i$$

$d_i = \sum_{j=1}^{N} a_{ij}$ Row sum= in-degree N_i In-neighbors of node i

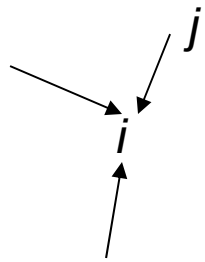


$d_i^o = \sum_{j=1}^{N} a_{ji}$ Col sum= out-degree N_o Out-neighbors of node i



Dynamic Graph- the Distributed Structure of Control

Each node has an associated state $\dot{x}_i = u_i$



Standard local voting protocol $u_i = \sum_{j \in N_i} a_{ij}(x_j - x_i)$

$$u_i = -x_i \sum_{j \in N_i} a_{ij} + \sum_{j \in N_i} a_{ij} x_j = -d_i x_i + \begin{bmatrix} a_{i1} & \cdots & a_{iN} \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_N \end{bmatrix}$$

x

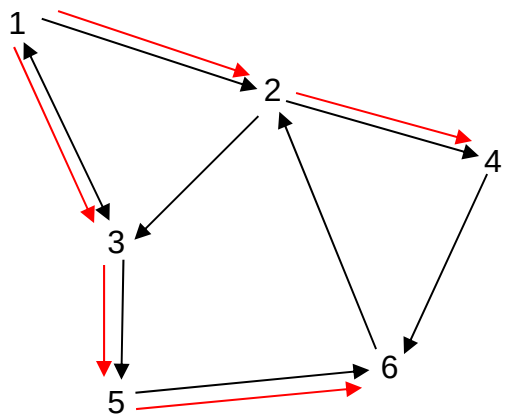
$$u = \begin{bmatrix} u_1 \\ \vdots \\ u_N \end{bmatrix} \qquad D = \begin{bmatrix} d_1 & & \\ & \ddots & \\ & & d_N \end{bmatrix} \qquad A = [a_{ij}]$$

$$u = -Dx + Ax = -(D - A)x = -Lx$$

L=D-A = graph Laplacian matrix

$\dot{x} = -Lx$ Closed-loop dynamics

If x is an n-vector then $\dot{x} = -(L \otimes I_n)x$



Theorem. Graph contains a spanning tree iff e-val of *L* at $\lambda_1 = 0$ is simple.

Then $\lambda_2 > 0$

Then -*L* has one e-val at zero and all the rest stable

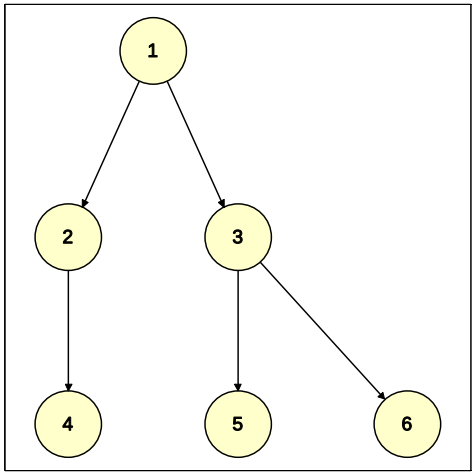
Graph strongly connected implies exists a spanning tree

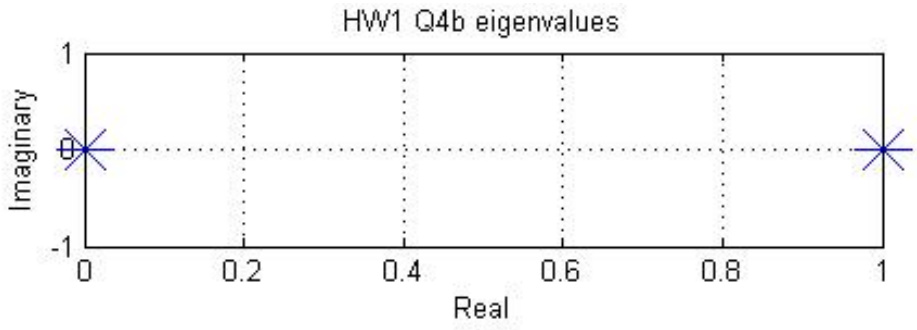
Then, all nodes synchronize

$\dot{x} = -Lx$ Closed-loop dynamics

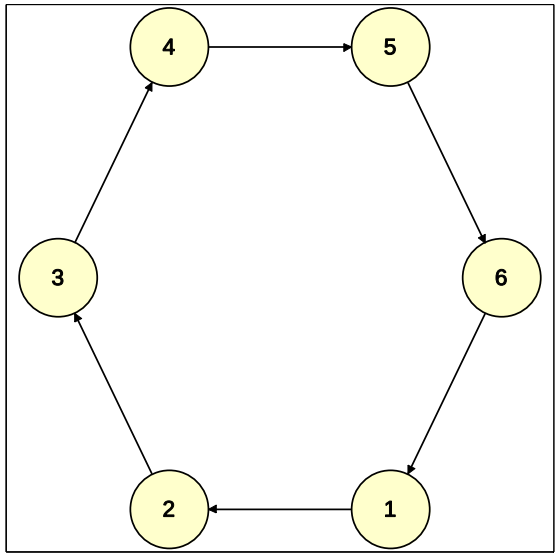
Graph Eigenvalues for Different Communication Topologies

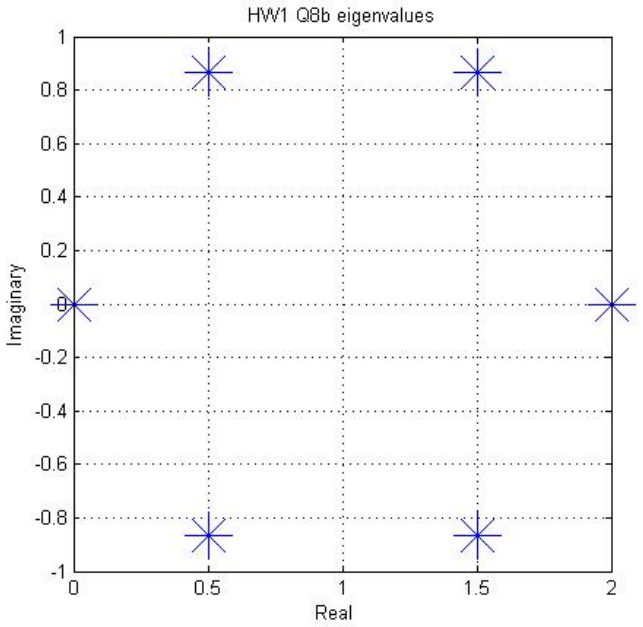
Directed Tree-
Chain of
command





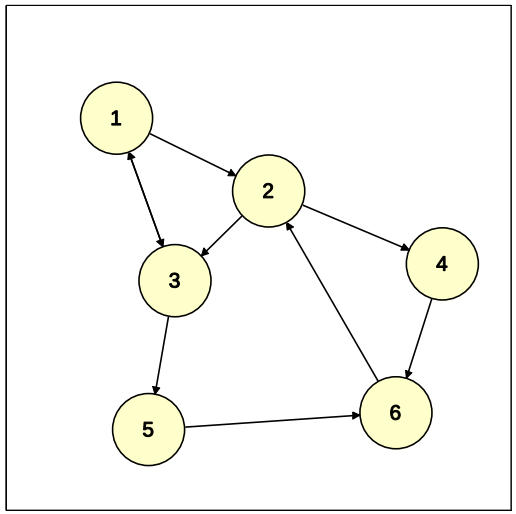
Directed Ring-
Gossip network
OSCILLATIONS

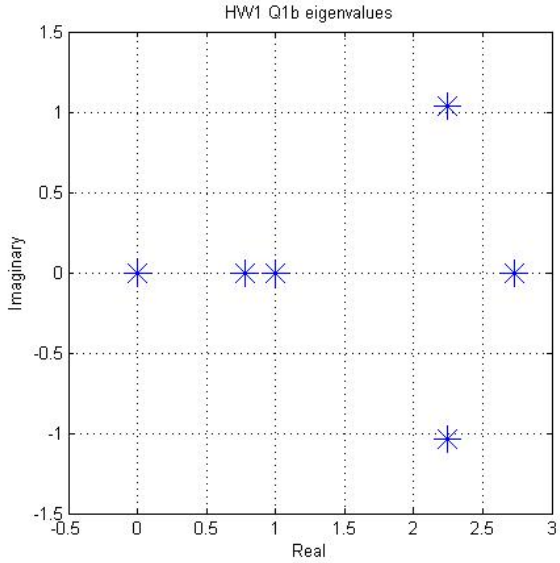




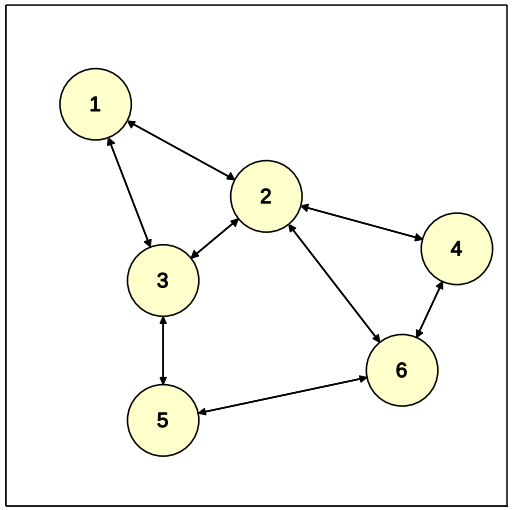
Graph Eigenvalues for Different Communication Topologies

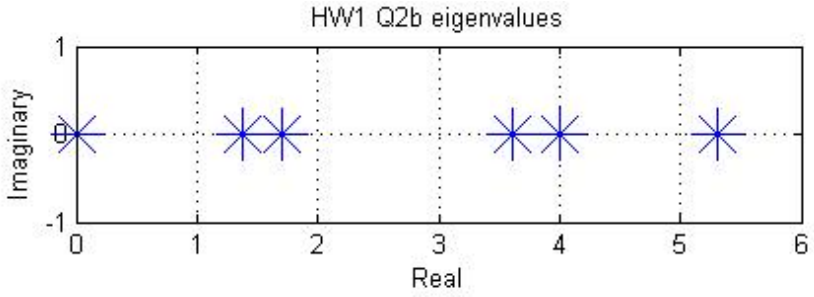
Directed graph-
Better conditioned





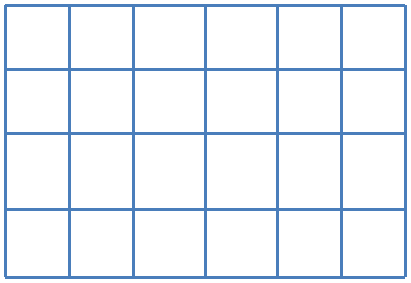
Undirected graph-
More ill-
conditioned



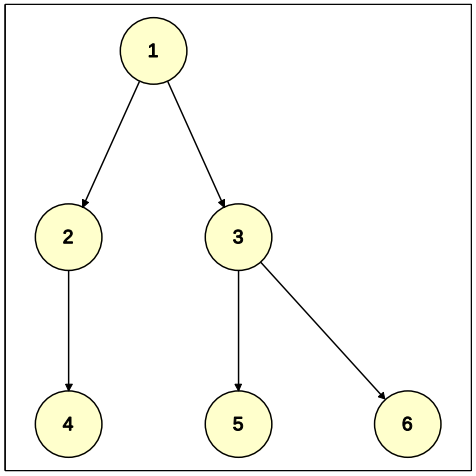


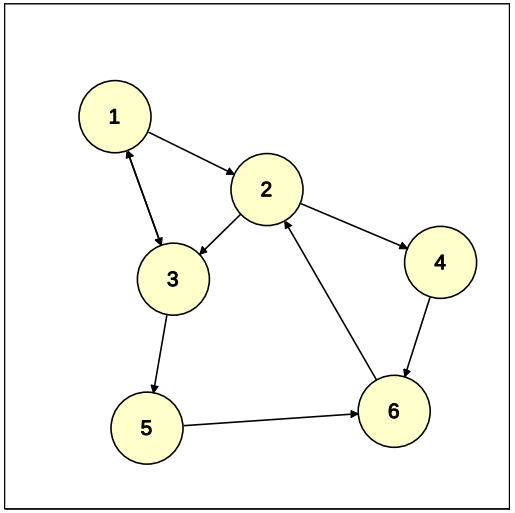
Synchronization on Good Graphs

Chris Elliott fast video



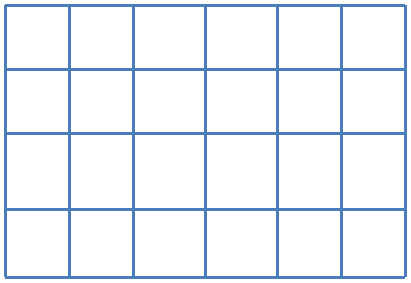
Regular mesh

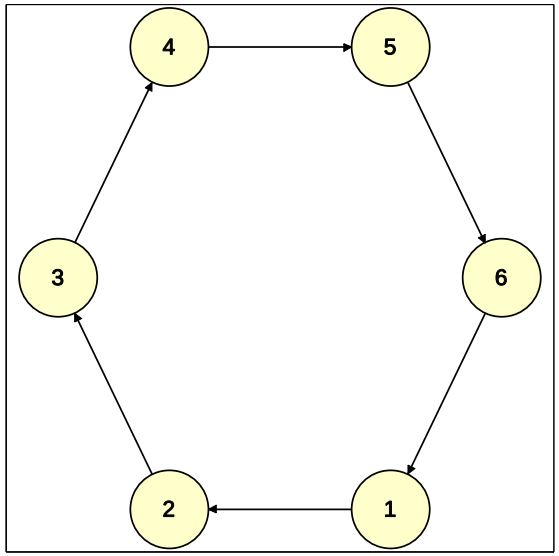




Synchronization on Gossip Rings

Chris Elliott weird video





Controlled Consensus: Cooperative Tracker

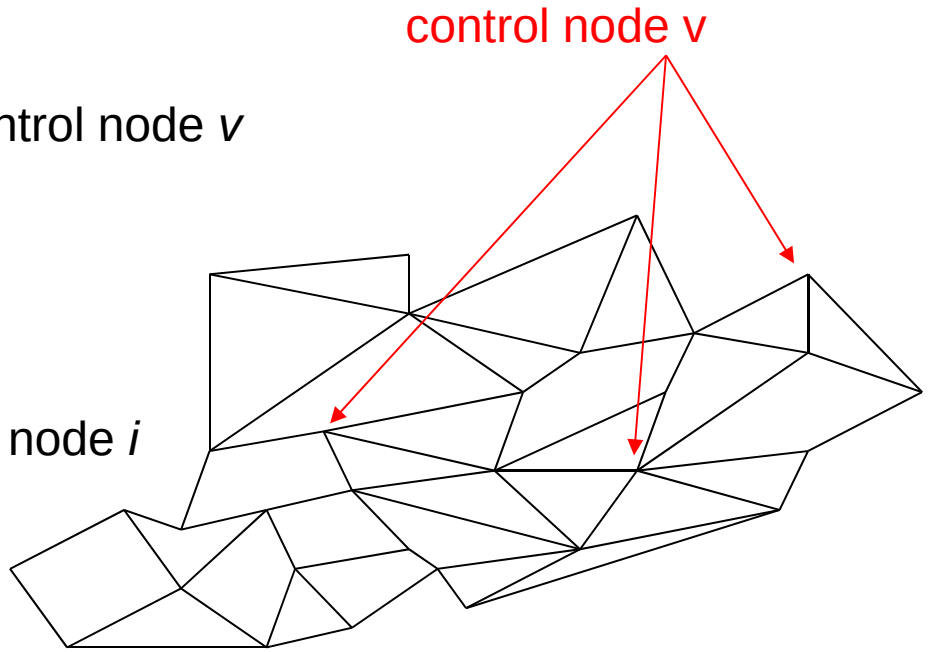
Node state $\dot{x}_i = u_i$

Distributed Local voting protocol with control node *v*

$$u_i = \sum_{j \in N_i} a_{ij}(x_j - x_i) \quad + g_i(v - x_i)$$

Local Neighborhood Tracking Error

$g_i \neq 0$ If control v is in the neighborhood of node *i*



$$u_i = -\sum_{j \in \bar{N}_i} a_{ij} x_i + \sum_{j \in N_i} a_{ij} x_j \quad + g_i v$$

$$\dot{x} = -(L+G)x + G\underline{1}v \qquad G = diag\{g_i\}$$

Theorem. Let graph have a spanning tree and $g_i \neq 0$ for at least one root node.
Then *L+G* is nonsingular with all e-vals positive
and *-(L+G)* is asymptotically stable

Ron Chen – pinning control



Kung Tz 500 BC

Confucius

孔子

Tian xia da tong
Harmony under heaven

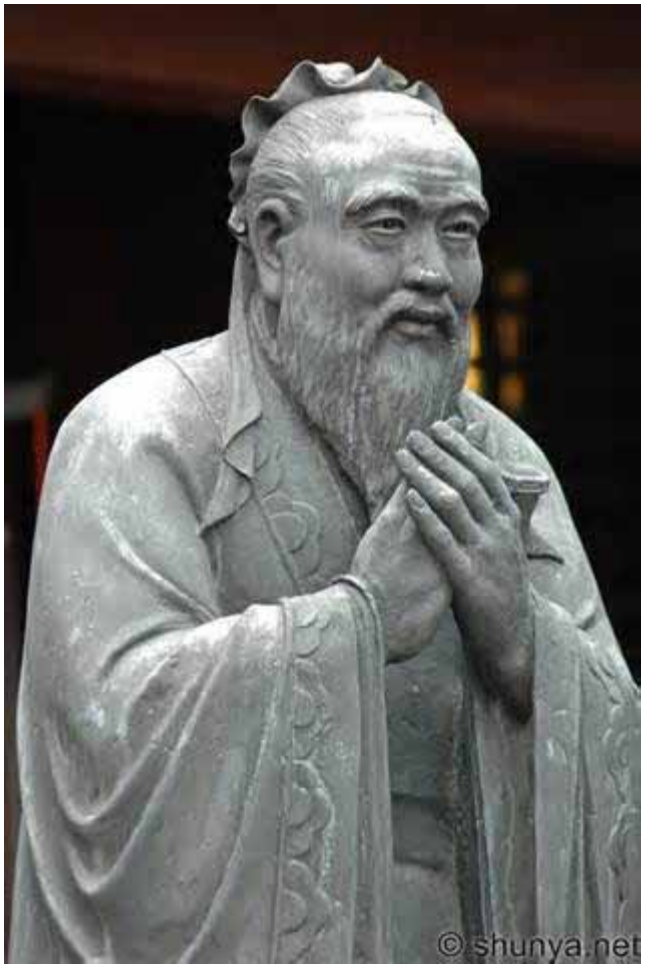
Archery
Chariot driving

Music
Rites and Rituals

Poetry
Mathematics



Man's relations to
- Family
- Friends
- Society
- Nation
- Emperor
- Ancestors



124 BC - Han Imperial University in Chang-an

What is a micro-grid?

- Micro-grid is a small-scale power system that provides the power for a group of consumers.
- Micro-grid enables local power support for local and critical loads.
- Micro-grid has the ability to work in both grid-connected and islanded modes.
- Micro-grid facilitates the integration of Distributed Energy Resources (DER).

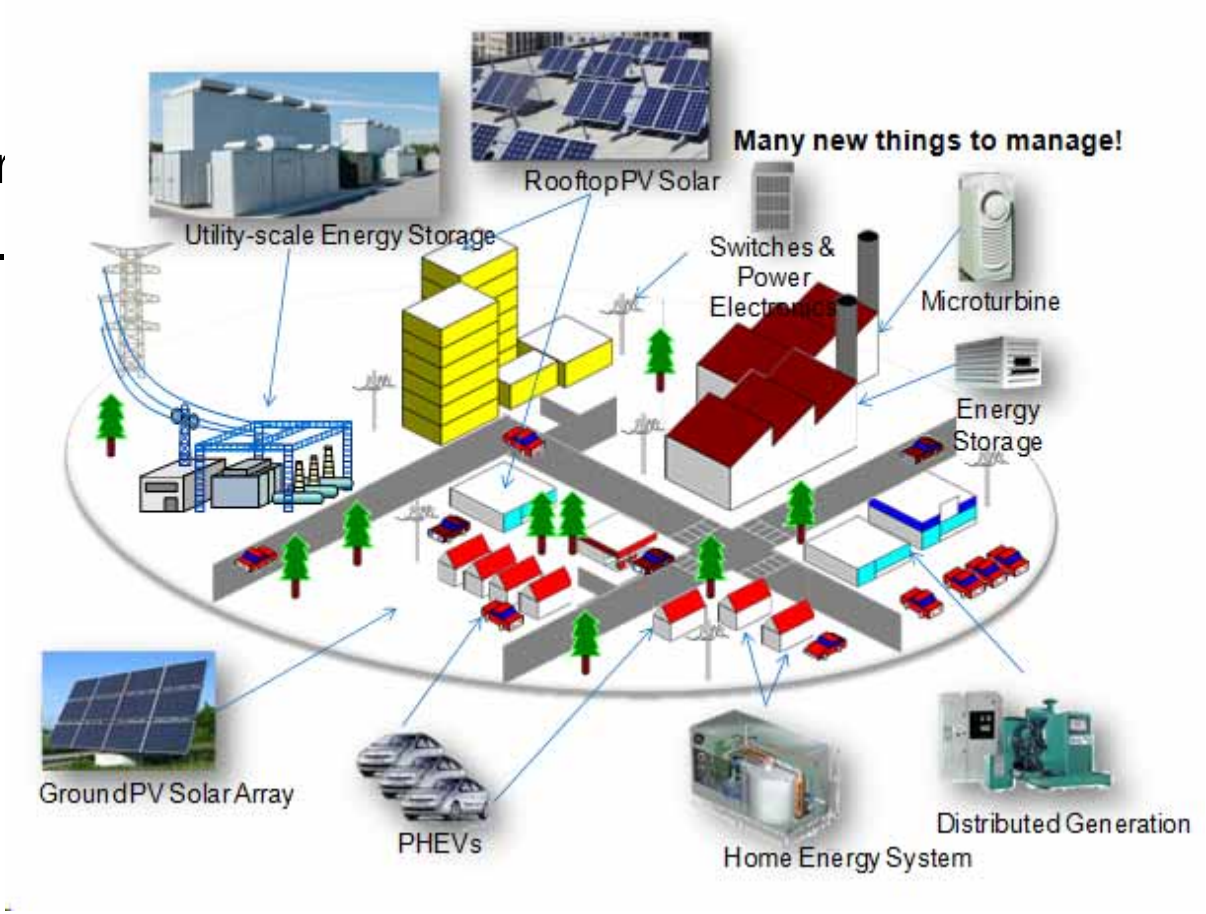
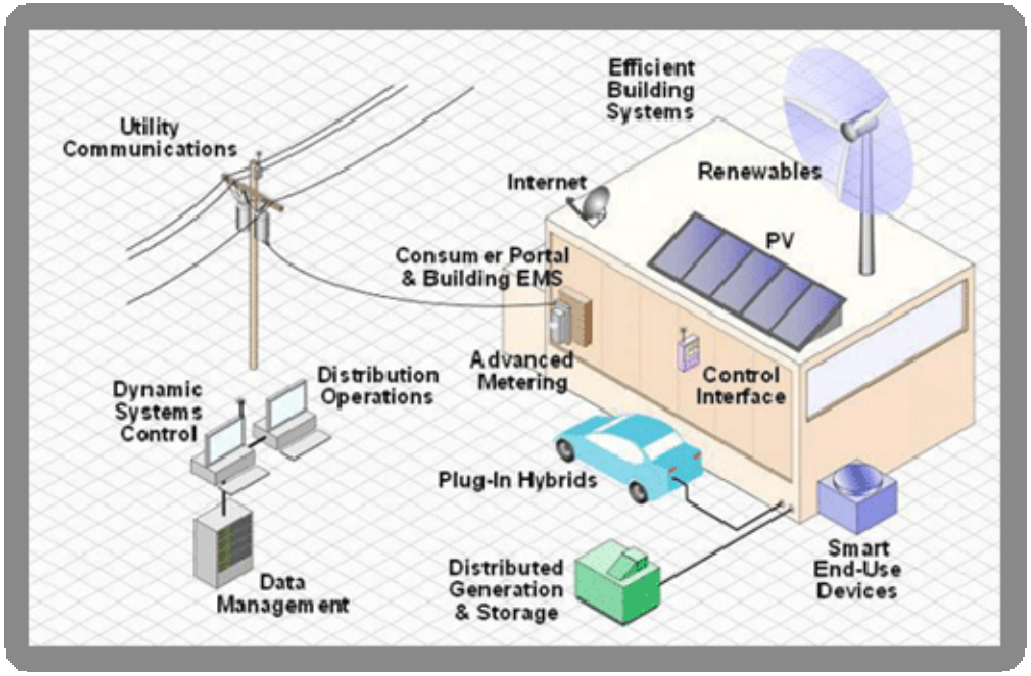


Photo from: http://www.horizonenergygroup.com

An introduction to micro-grids: Micro-grid applications

- The main building block of smart-grids
- Rural plants
- Business buildings, hospitals, and factories

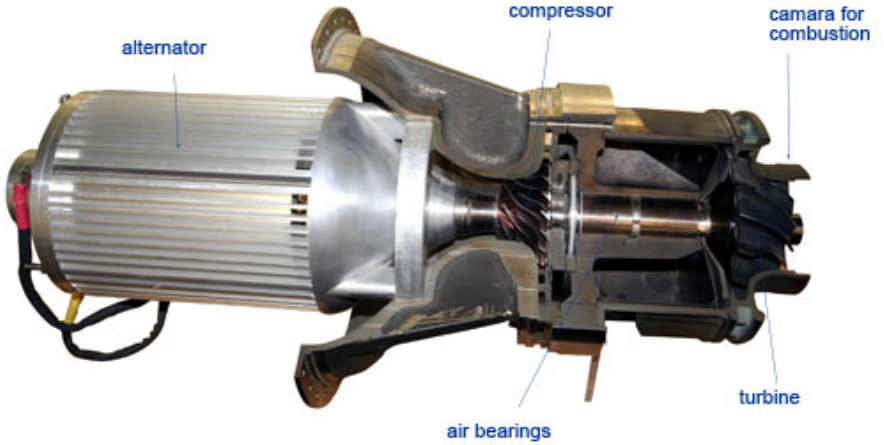




Smart-grid photo from: http://www.sustainable-sphere.com

Distributed Generators (DG)
Distributed Energy Resources (DER)

- Non-renewables
  - Internal combustion engine
  - Micro-turbines
  - Fuel cells
- Renewables
  - Photovoltaic
  - Wind
  - Hydroelectric
  - Biomass





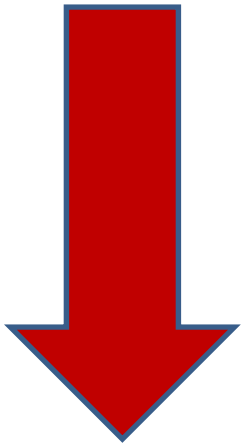


Micro-grid Advantages

- Micro-grid provides high quality and reliable power to the critical consumers
- During main grid disturbances, micro-grid can quickly disconnect form the main grid and provide reliable power for its local loads
- DGs can be simply installed close to the loads which significantly reduces the power transmission line losses
- By using renewable energy resources, a micro-grid reduces CO2 emissions

An introduction to micro-grids: Micro-grid Objectives

- Voltage and frequency synchronization for both grid-connected and islanded operating modes
- Proper load sharing and DG coordination
- Power flow control between the microgrid and the main grid
- Optimizing the microgrid operating cost



Hierarchical control structure

Micro-grid Control Challenges

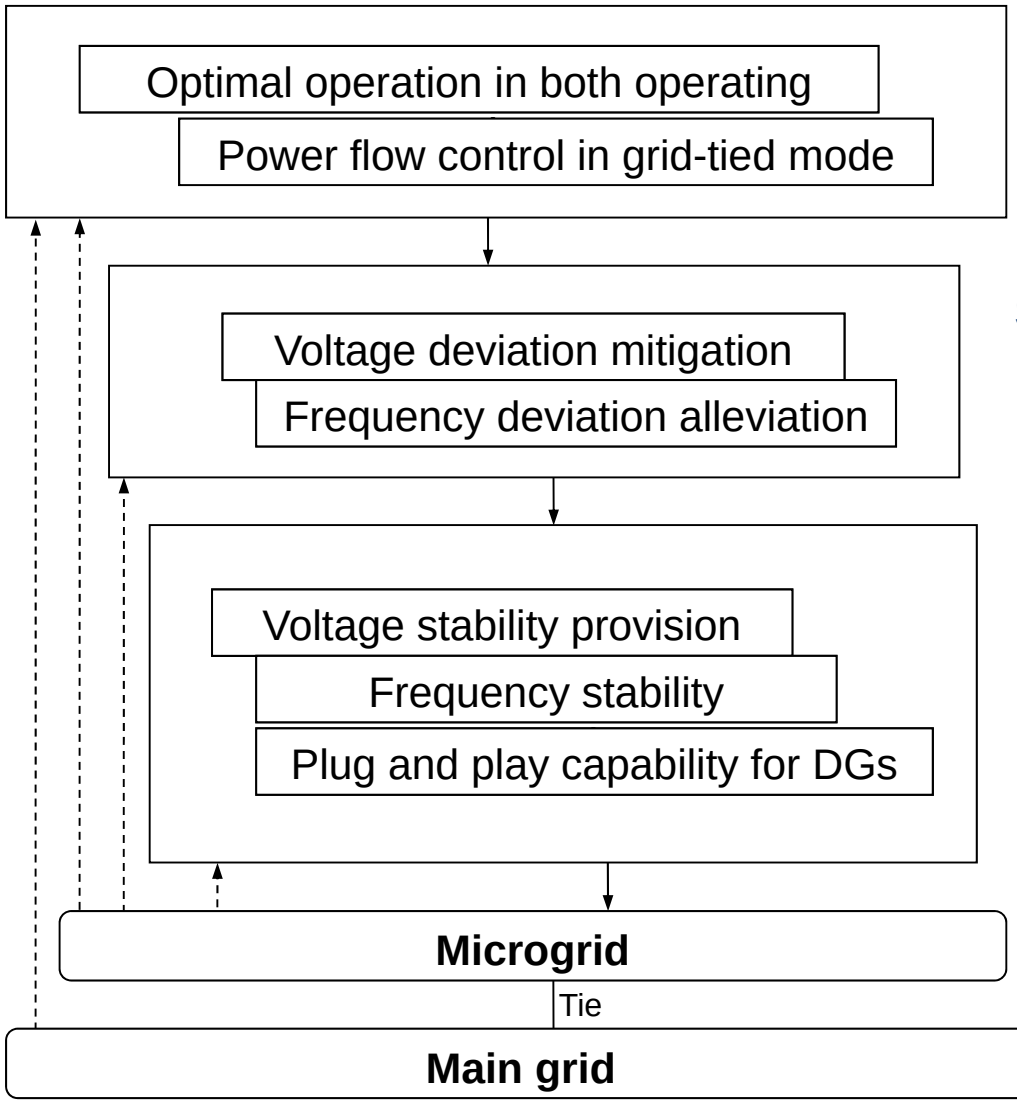
- In Grid-connected mode, the main grid has rotating synchronous generators that provide a frequency reference
- In Grid-connected mode, the main grid provides voltage support and power quality

- During grid disturbances, micro-grid goes to islanded mode to provide the power for its local loads

- In islanded mode, there is no frequency reference
- In islanded mode, microgrid controller must provide voltage support and power quality

Bidram, A., & Davoudi, A. (2012). Hierarchical structure of microgrids control system. *IEEE Transactions on Smart Grid*, vol. 3, pp. 1963-1976, Dec 2012.

Micro-grid Hierarchical Control Structure



Tertiary Control

Secondary Control

Do coop. ctrl. here to Synchronize frequency and voltage

Primary Control

Maintains Stability

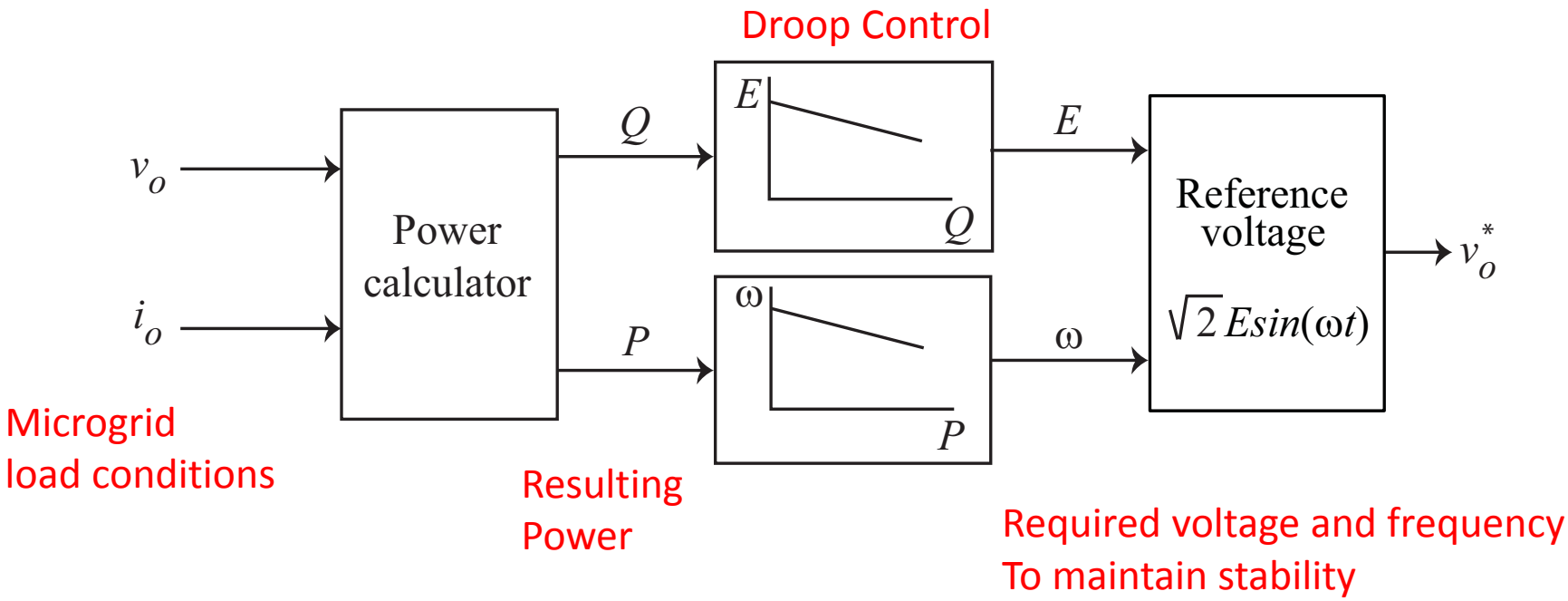
Micro-grid Primary Control

- **Primary control**: The primary control maintains voltage and frequency stability
- **Conventional primary control**: Droop techniques

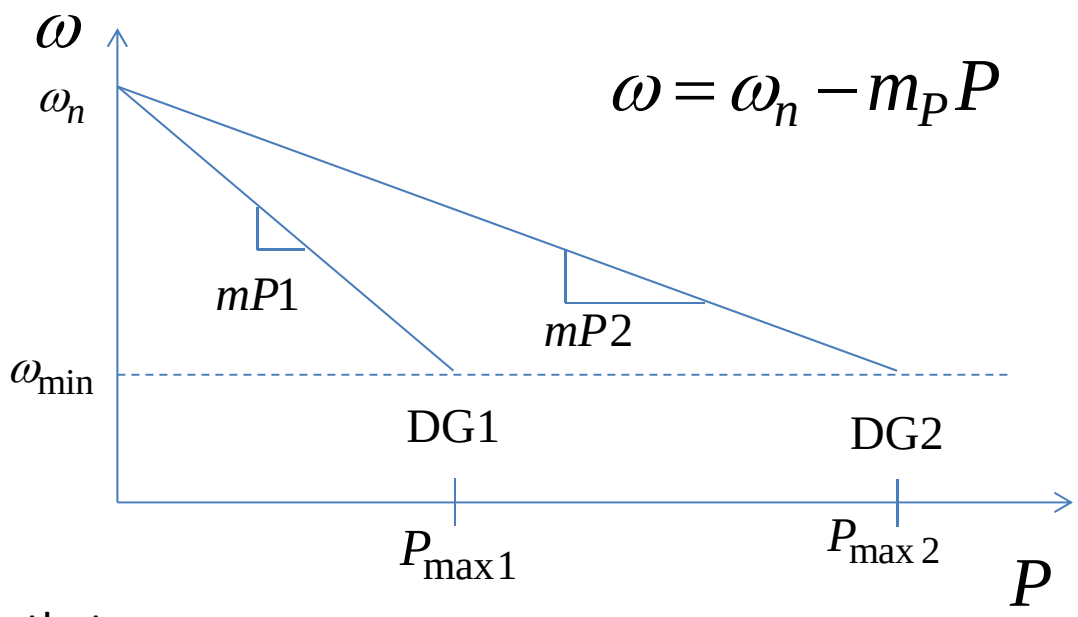
$$\begin{cases} \omega = \omega_n - m_P P \\ E \equiv v_{mag} = V_n - n_Q Q \end{cases}$$

$$m_{P1} P_{\max 1} = \cdots = m_{PN} P_{\max N}$$

$$n_{Q1} Q_{\max 1} = \cdots = n_{QN} Q_{\max N}$$



Design of Droop Control Parameters



Pick slopes so that

$$mP1 = (\omega_n - \omega_{\min}) / \mathrm{P}_{\max 1} \qquad mP2 = (\omega_n - \omega_{\min}) / \mathrm{P}_{\max 2}$$

Then

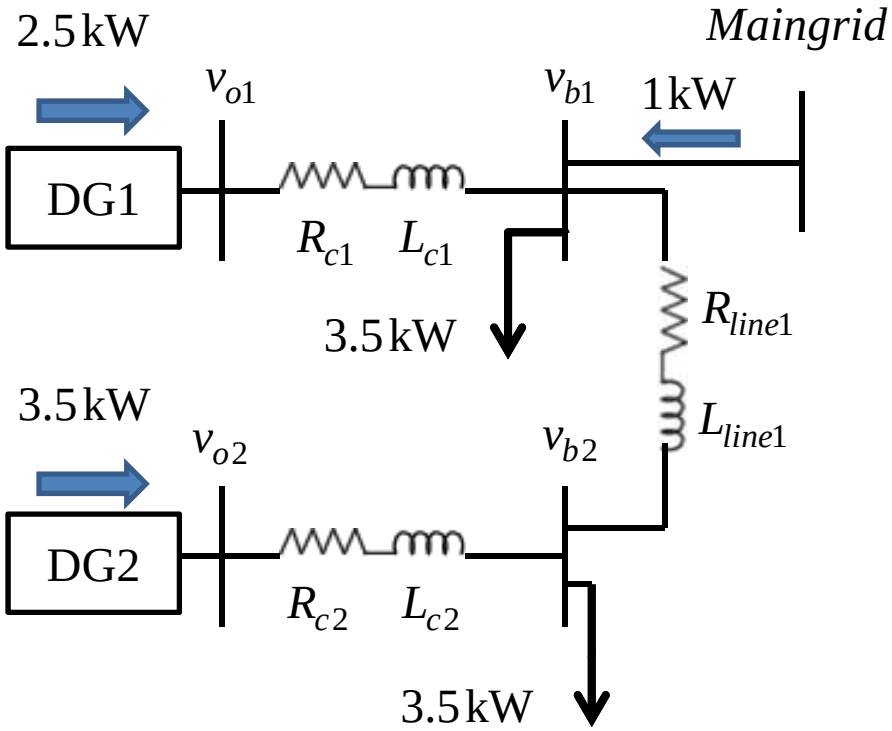
$$m_{P1} P_{\max 1} = \cdots = m_{PN} P_{\max N}$$

Balanced Load sharing

Micro-grid primary control

1. Connected to Main Power Grid
   Load is 7kW
   Grid supplies 1kW

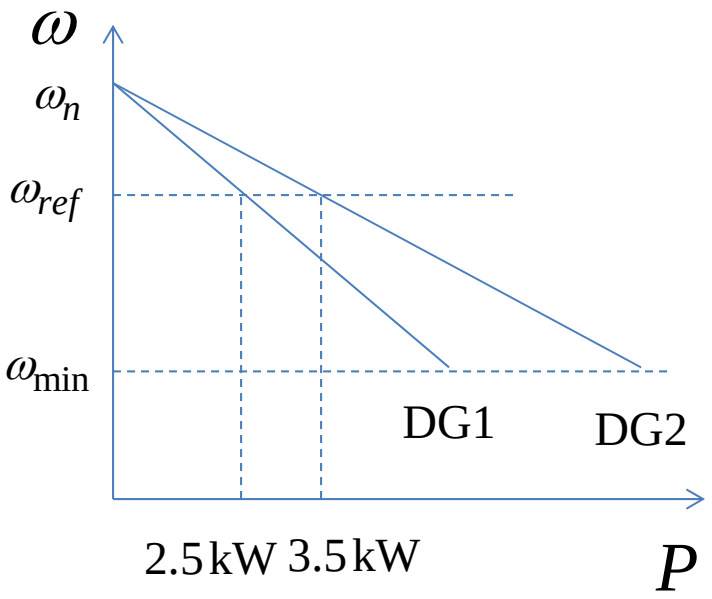
- **Primary control** (frequency droop)
- Before islanding occurs



DG1 nominal power, Pmax1= 4 kW

DG2 nominal power, Pmax2= 6 kW

$$m_{P1}P_{\max 1} = m_{P2}P_{\max 2}$$



Micro-grid Primary Control

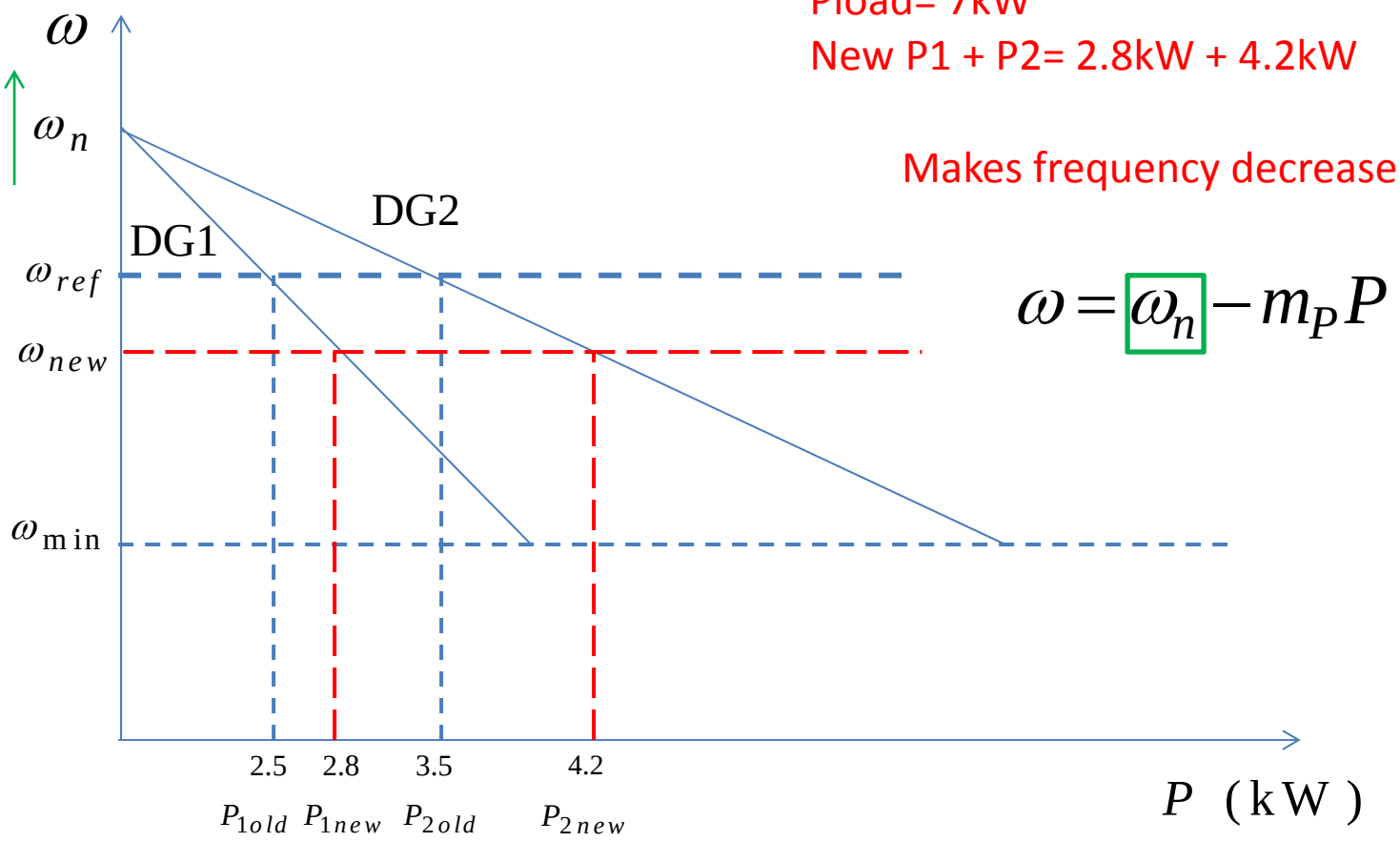
2. After Islanding

DGs must make up an extra 1kW

Pload= 7kW

New P1 + P2= 2.8kW + 4.2kW

Makes frequency decrease



$$\omega = \omega_n - m_P P$$

Increase ω_n to restore frequency to ref value

Secondary Frequency Control

New Secondary Control Input for Frequency Synchronization

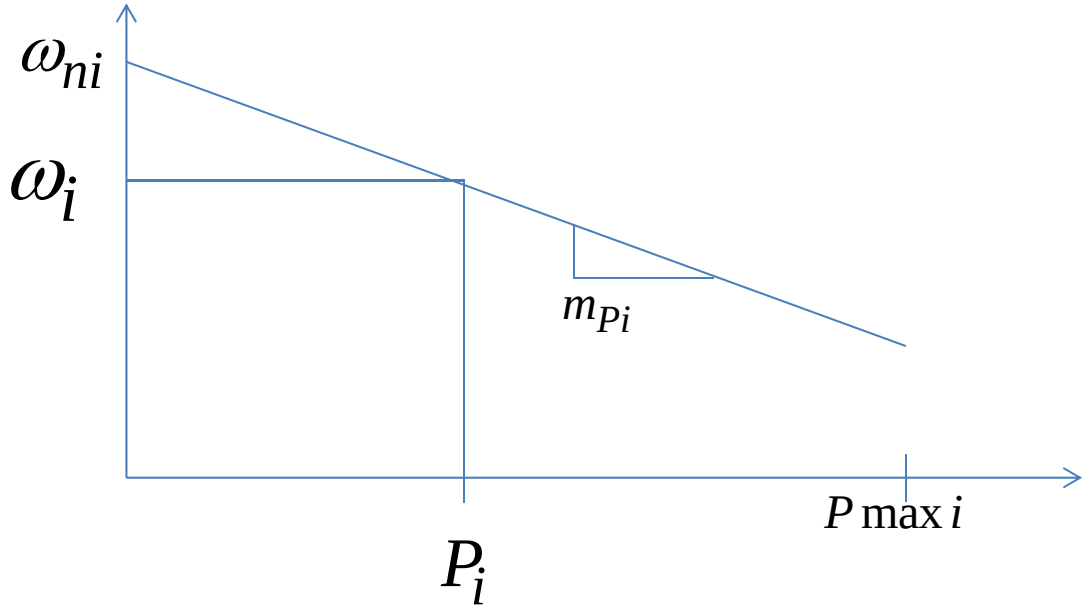
$$\omega_i = \omega_{ni} - m_{Pi} P_i$$

Secondary Control input

Prescribed frequency e.g. 60 Hz

Change ω_{ni}

To synchronize ω_i



Existing power conditions in the microgrid

Micro-grid secondary control

- **Secondary control**: The secondary control restores the voltage and frequency of the micro-grid to their nominal value.

Conventional Secondary control implementation:

- Centralized structure
  - Low reliability
  - Requires a Central control authority
  - Requires too many communication links

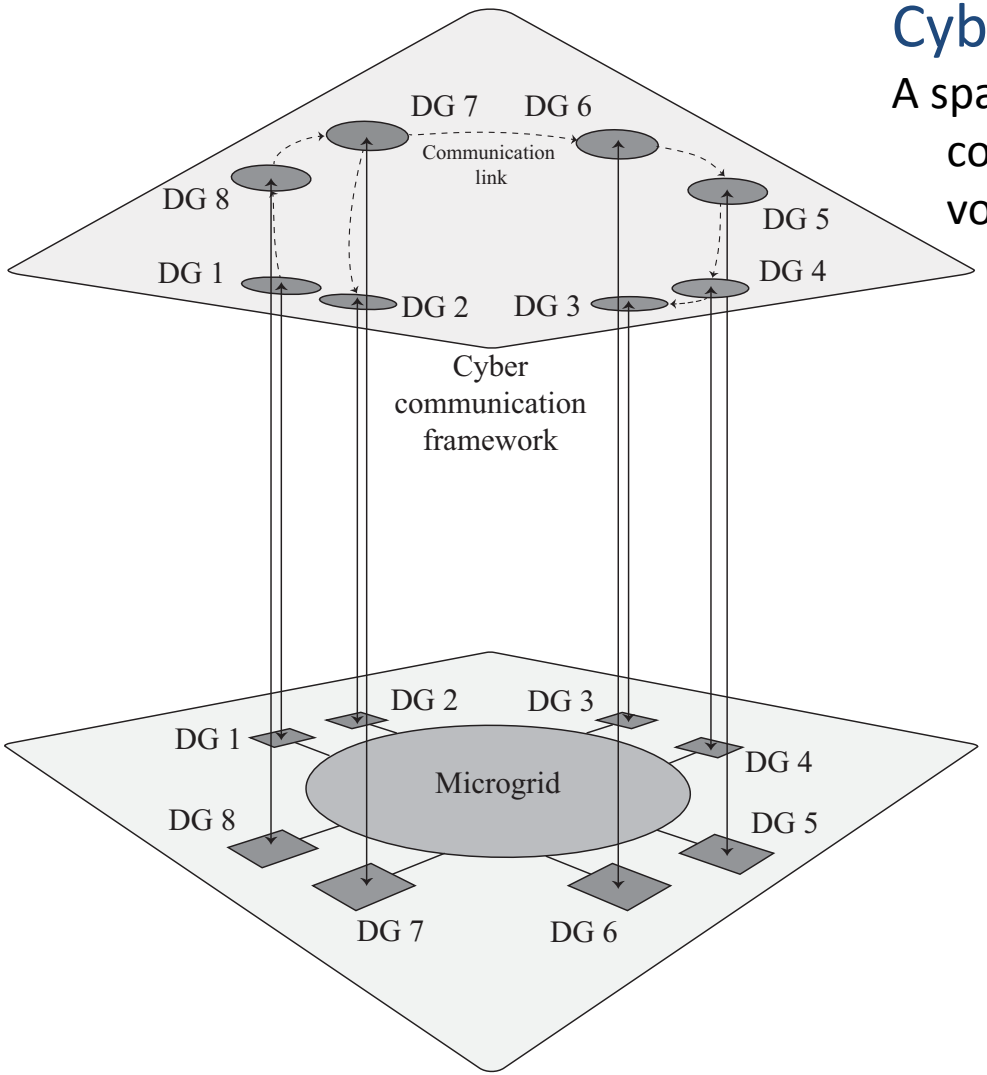
$$\begin{cases} \delta V_n = K_{PE}(v_{ref} - v_{mag}) + K_{IE}\int (v_{ref} - v_{mag})dt \\ \delta \omega_n = K_{P\omega}(\omega_{ref} - \omega) + K_{I\omega}\int (\omega_{ref} - \omega)dt \end{cases}$$

- We want to develop a new Distributed Control structure
  - Highly reliable
  - Uses sparse communication network


Photo by

Micro-grid secondary control: New Distributed CPS structure

Work of Ali Bidram
With Dr. A. Davoudi



Cyber layer

A sparse, efficient communication network to allow cooperative control for synchronization of voltage and frequency

Cyber Physical System (CPS)

Physical Layer

The interconnect structure of the power grid

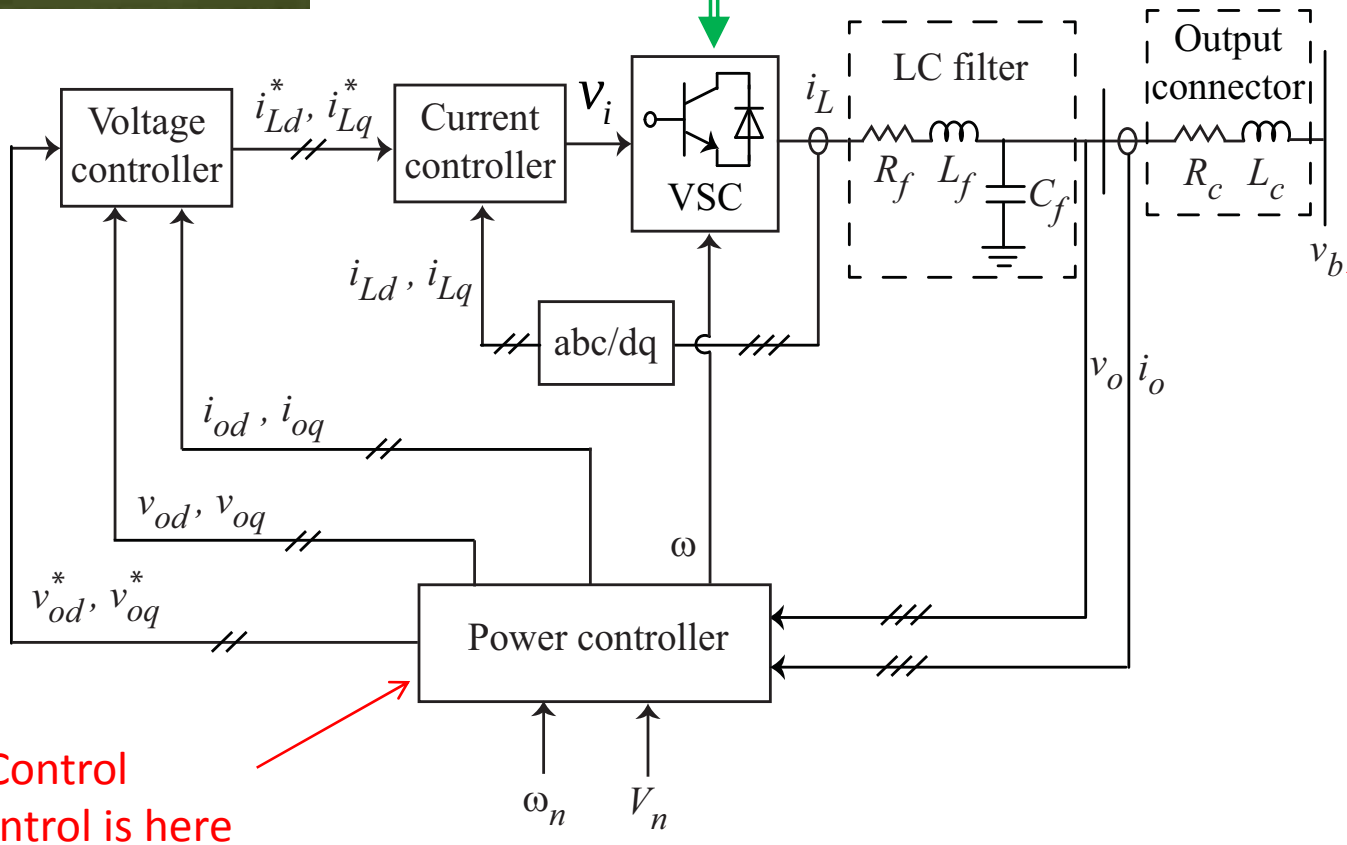
Dynamical model of a DG

Primary Control Structure



Renewable DER
Provides DC voltage

VSC- Voltage source converter
Power electronics



Microgird
Network
Load
disturbances

Primary Control
Droop control is here

Given load conditions v_0, i_0 - pick $\omega, v_{od}^*, v_{oq}^*$ using Droop to maintain stability

Pogaku, N., Prodanovic, M., & Green, T. C. (2007). Modeling, analysis and testing of autonomous operation of an inverter-based microgrid. *IEEE Transactions on Power Electronics*, *22*(2), 613–625.

Dynamical model of a DG

$$\begin{cases} \dot{\mathbf{x}}_i = \mathbf{f}_i(\mathbf{x}_i) + \mathbf{k}_i(\mathbf{x}_i)\mathbf{D}_i + \mathbf{g}_i(\mathbf{x}_i)u_i \\ y_i = h_i(\mathbf{x}_i) + d_i u_i \end{cases}$$

Heterogeneous agent dynamics

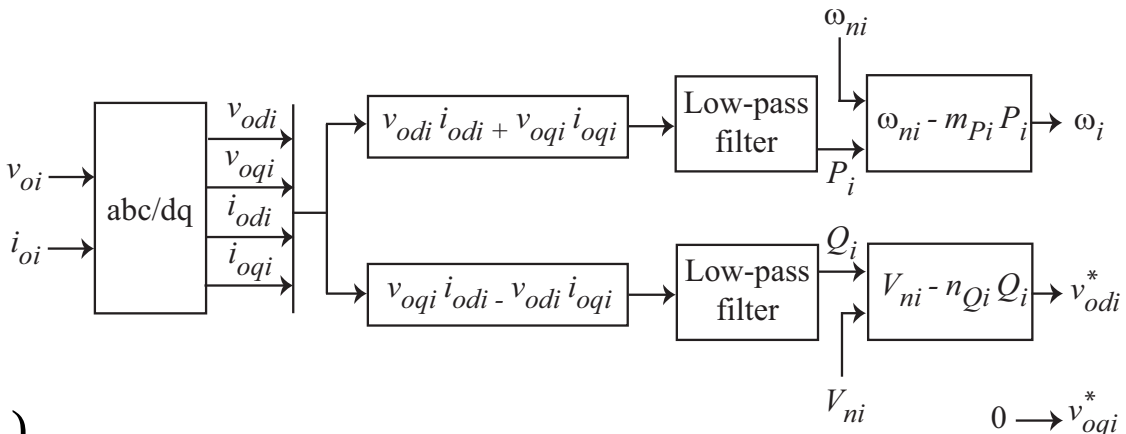
$$\mathbf{x}_i \in \mathbb{R}^{13} \qquad \mathbf{x}_i = [\delta_i \quad P_i \quad Q_i \quad \phi_{di} \quad \phi_{qi} \quad \gamma_{di} \quad \gamma_{qi} \quad i_{ldi} \quad i_{lqi} \quad v_{odi} \quad v_{oqi} \quad i_{odi} \quad i_{oqi}]^T$$

Power controller dynamics:

$$\dot{\delta}_i = \omega_i - \omega_{com}$$

$$\dot{P}_i = -\omega_{ci} P_i + \omega_{ci}(v_{odi} i_{odi} + v_{oqi} i_{oqi})$$

$$\dot{Q}_i = -\omega_{ci} Q_i + \omega_{ci}(v_{oqi} i_{odi} - v_{odi} i_{oqi})$$



Droop control is here

Dynamical model of a DG

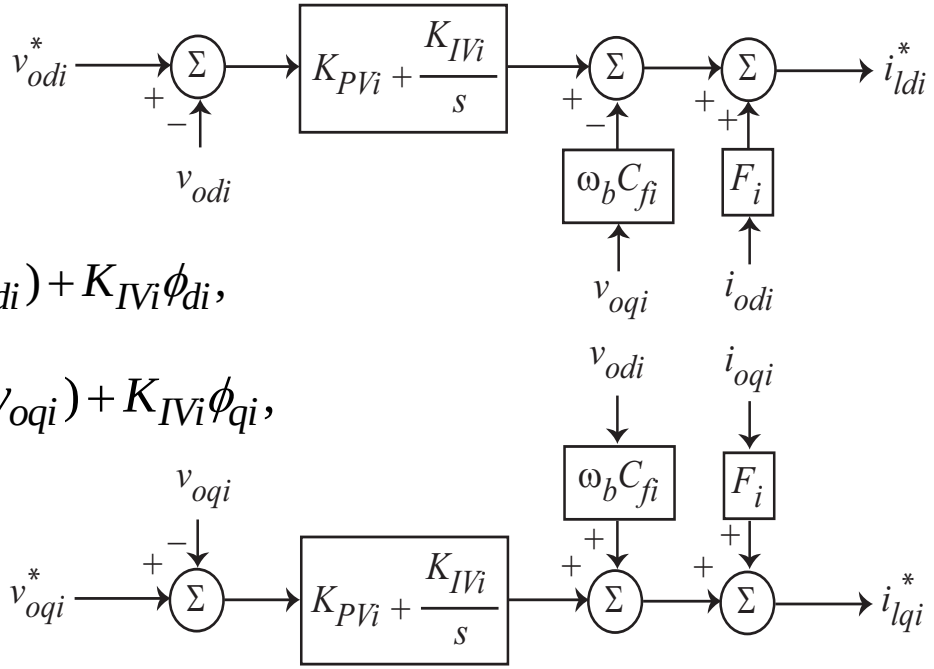
Voltage controller dynamics

$$\dot{\phi}_{di} = v^*_{odi} - v_{odi},$$

$$\dot{\phi}_{qi} = v^*_{oqi} - v_{oqi},$$

$$i^*_{ldi} = F_i i_{odi} - \omega_b C_{fi} v_{oqi} + K_{PVi}(v^*_{odi} - v_{odi}) + K_{IVi}\phi_{di},$$

$$i^*_{lqi} = F_i i_{oqi} + \omega_b C_{fi} v_{odi} + K_{PVi}(v^*_{oqi} - v_{oqi}) + K_{IVi}\phi_{qi},$$



Dynamical model of a DG

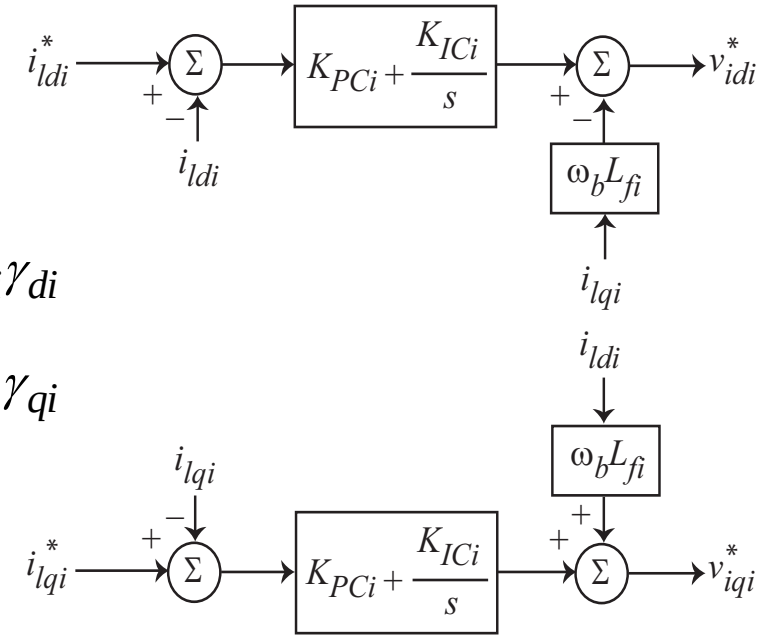
Current controller dynamics

$$\dot{\gamma}_{di} = i^*_{ldi} - i_{ldi}$$

$$\dot{\gamma}_{qi} = i^*_{lqi} - i_{lqi}$$

$$v^*_{idi} = -\omega_b L_{fi} i_{lqi} + K_{PCi}(i^*_{ldi} - i_{ldi}) + K_{ICi}\gamma_{di}$$

$$v^*_{iqi} = \omega_b L_{fi} i_{ldi} + K_{PCi}(i^*_{lqi} - i_{lqi}) + K_{ICi}\gamma_{qi}$$



Dynamical model of a DG

Output filter dynamics

$$\dot{i}_{ldi} = -\frac{R_{fi}}{L_{fi}} i_{ldi} + \omega_i i_{lqi} + \frac{1}{L_{fi}} v_{idi} - \frac{1}{L_{fi}} v_{odi}$$

$$\dot{i}_{lqi} = -\frac{R_{fi}}{L_{fi}} i_{lqi} - \omega_i i_{ldi} + \frac{1}{L_{fi}} v_{iqi} - \frac{1}{L_{fi}} v_{oqi}$$

$$\dot{v}_{odi} = \omega_i v_{oqi} + \frac{1}{C_{fi}} i_{ldi} - \frac{1}{C_{fi}} i_{odi}$$

$$\dot{v}_{oqi} = -\omega_i v_{odi} + \frac{1}{C_{fi}} i_{lqi} - \frac{1}{C_{fi}} i_{oqi}$$

Output connector dynamics

$$\dot{i}_{odi} = -\frac{R_{ci}}{L_{ci}} i_{odi} + \omega_i i_{oqi} + \frac{1}{L_{ci}} v_{odi} - \frac{1}{L_{ci}} v_{bdi}$$

$$\dot{i}_{oqi} = -\frac{R_{ci}}{L_{ci}} i_{oqi} - \omega_i i_{odi} + \frac{1}{L_{ci}} v_{oqi} - \frac{1}{L_{ci}} v_{bqi}$$

Depends on microgrid conditions and loads

Voltage disturbances

The nonlinear dynamics of the i^{th} DG, while neglecting the fast dynamics of voltage and current controllers

$$
\begin{cases}
\dot{x}_{i,1} = \omega_{ni} - m_{Pi} x_{i,2} - \omega_{com} \\
\dot{x}_{i,2} = \omega_{ci}(x_{i,6}x_{i,8} + x_{i,7}x_{i,9} - x_{i,2}) \\
\dot{x}_{i,3} = \omega_{ci}(x_{i,6}x_{i,9} - x_{i,7}x_{i,8} - x_{i,3}) \\
\dot{x}_{i,4} = \dfrac{-r_{fi}}{L_{fi}} x_{i,4} + \omega_{com} x_{i,5} + \dfrac{V_{ni} - n_{Qi} x_{i,3} - x_{i,6}}{L_{fi}} \\
\dot{x}_{i,5} = \dfrac{-r_{fi}}{L_{fi}} x_{i,5} - \omega_{com} x_{i,4} - \dfrac{x_{i,7}}{L_{fi}} \\
\dot{x}_{i,6} = \omega_{com} x_{i,7} + \dfrac{x_{i,4} - x_{i,8}}{C_{fi}} \\
\dot{x}_{i,7} = -\omega_{com} x_{i,6} + \dfrac{x_{i,5} - x_{i,9}}{C_{fi}} \\
\dot{x}_{i,8} = \dfrac{-r_{ci}}{L_{ci}} x_{i,8} + \omega_{com} x_{i,9} + \dfrac{x_{i,6} - v_{bdi}}{L_{ci}} \\
\dot{x}_{i,9} = \dfrac{-r_{ci}}{L_{ci}} x_{i,9} - \omega_{com} x_{i,8} + \dfrac{x_{i,7} - v_{bqi}}{L_{ci}}
\end{cases}
$$

$$
x_i = [\alpha_i \quad P_i \quad Q_i \quad i_{Ldi} \quad i_{Lqi} \quad v_{odi} \quad v_{oqi} \quad i_{odi} \quad i_{oqi}]^T .
$$

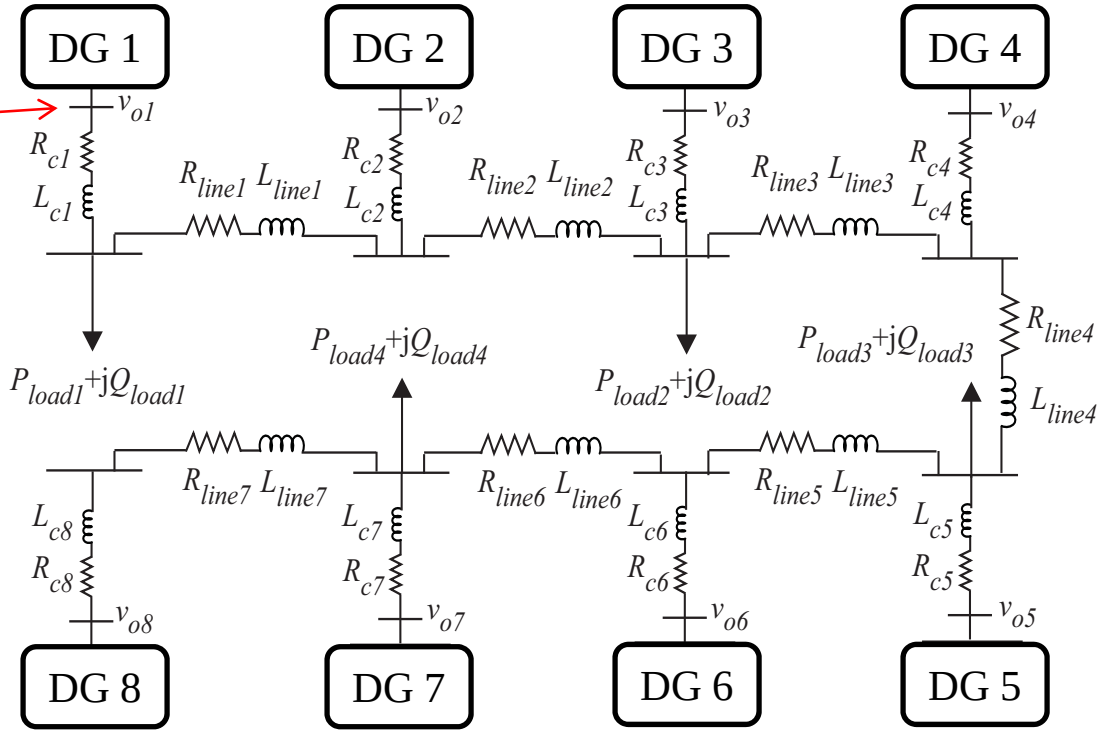
DG Microgrid Model and Synchronization Control Objectives

Heterogeneous Agent Dynamics – different dynamics

$$\begin{cases} \dot{\mathbf{x}}_i = \mathbf{f}_i(\mathbf{x}_i) + \mathbf{k}_i(\mathbf{x}_i)\mathbf{D}_i + \mathbf{g}_i(\mathbf{x}_i)u_i \\ y_i = h_i(\mathbf{x}_i) + d_i u_i \end{cases} \qquad \mathbf{x}_i \in \mathbb{R}^{13}$$

Synchronization in Microgrid of Interconnected DG

Voltage synchronization



Frequency synchronization

$$y_i = \omega_i = \omega_{ni} - m_{Pi} P_i$$

Voltage synchronization (per unit)

$$E \equiv v_{mag} = V_n - n_Q Q$$

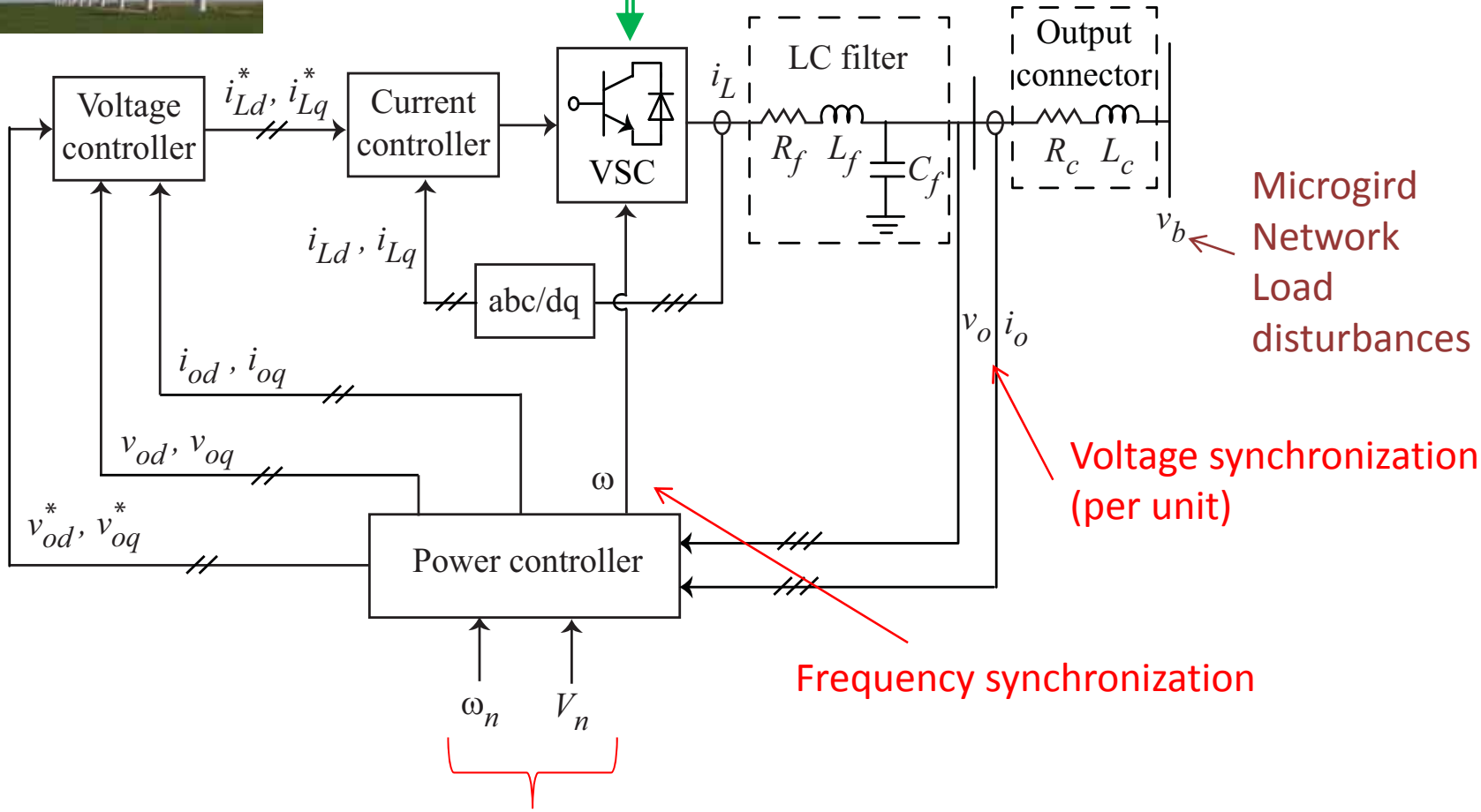
$$v_{o,magi} = \sqrt{v_{odi}^{\,2} + v_{oqi}^{\,2}}$$

Secondary Control Synchronization Objectives



Renewable DER
Provides DC voltage

VSC- Voltage source converter
Power electronics



Microgird
Network
Load
disturbances

Voltage synchronization
(per unit)

Frequency synchronization

Secondary Control Inputs
Change Droop control parameters to get synchronization

DG Microgrid Model and Synchronization Control Objectives

Heterogeneous Agent Dynamics – different dynamics

$$\begin{cases} \dot{\mathbf{x}}_i = \mathbf{f}_i(\mathbf{x}_i) + \mathbf{k}_i(\mathbf{x}_i)\mathbf{D}_i + \mathbf{g}_i(\mathbf{x}_i)u_i \\ y_i = h_i(\mathbf{x}_i) + d_i u_i \end{cases} \qquad \mathbf{x}_i \in \mathbb{R}^{13}$$

1. For secondary frequency control:

$$y_i = \omega_i = \omega_{ni} - m_{Pi}P_i$$
$$u_i = \omega_{ni} \quad \Rightarrow \quad d_i \neq 0$$

2. For secondary voltage control:

$$E \equiv v_{mag} = V_n - n_Q Q \quad \Rightarrow \quad \begin{matrix} y_i = v_{odi} \\ u_i = V_{ni} \end{matrix} \quad \Rightarrow \quad d_i = 0$$


1. Distributed secondary frequency control of micro-grids
2. Distributed secondary voltage control of micro-grids
Work of Ali Bidram
With Dr. A. Davoudi

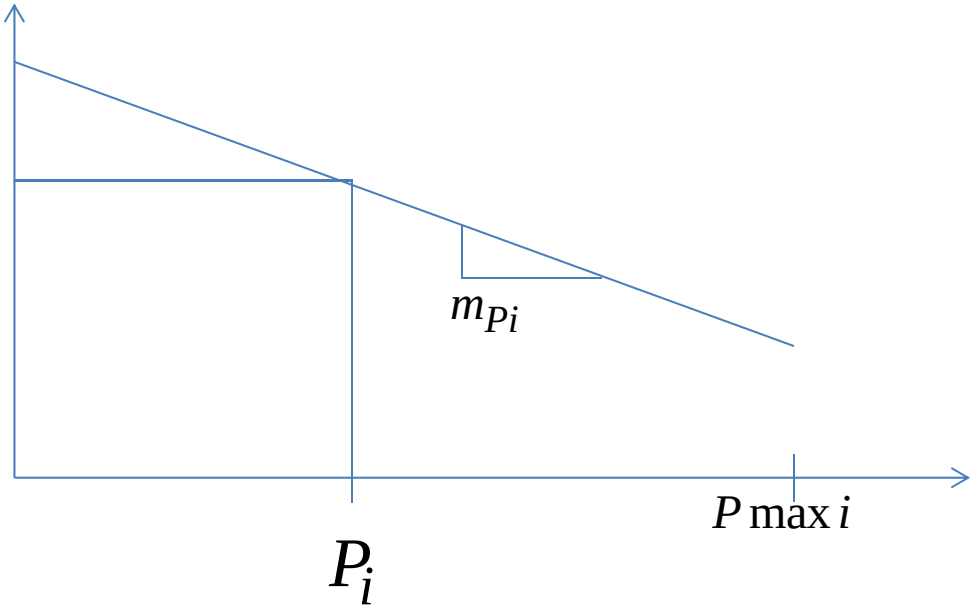
1. Secondary Frequency Control

New Secondary Control Input for Frequency Synchronization

$$\omega_i = \omega_{ni} - m_{Pi} P_i$$

Secondary Control input → ω_{ni}

Prescribed frequency ω_i



Existing power conditions in the microgrid

Change ω_{ni}

To synchronize ω_i

$$\omega_i \to \omega_{ref}, \quad \forall i$$

For example

$$\omega_{ref} = 50Hz$$

1. Secondary Frequency Control

Droop control relationship

$$\omega_i = \omega_{ni} - m_{Pi} P_i$$

Using input-output feedback linearization

$$\dot{\omega}_i = \dot{\omega}_{ni} - m_{Pi}\dot{P}_i \equiv u_{\omega i}$$

$$u_{\omega i} = -c_{\omega i} e_{\omega i}$$

$$e_{\omega i} = \sum_{j \in N_i} a_{ij}(\omega_i - \omega_j) + g_i(\omega_i - \omega_{ref})$$

Then $$\dot{\omega}_{ni} = m_{Pi}\dot{P}_i + u_{\omega i} = m_{Pi}\dot{P}_i - c_{\omega i}\sum_{j \in N_i} a_{ij}(\omega_i - \omega_j) + g_i(\omega_i - \omega_{ref})$$

Theorem . Let the digraph of the communication network have a spanning tree and the pinning gain be nonzero for at least one DG placed on a root node.

Let the auxiliary control $u_{\omega i}$ be chosen as above.

Then, the global neighborhood error is asymptotically stable.
Moreover, the DG frequencies synchronize to ω_{ref}

1. Secondary Frequency Control

Proof. Lyapunov Functions for Cooperative Control on Graphs

Graph Laplacian Matrix $L = D - \mathcal{A}$

$$e_\omega = \begin{bmatrix} e_{\omega 1} & e_{\omega 2} & \cdots & e_{\omega N} \end{bmatrix}^T \in \mathbb{R}^N \quad e_\omega = (L+G)\left(\omega - \underline{\omega}_{ref}\right), \quad u_\omega = -c_\omega e_\omega \qquad c_\omega = diag\{c_{\omega i}\}$$

Lyapunov function: $V = \frac{1}{2} e_\omega^T P e_\omega$, where $P = diag\left\{ 1/w_i \right\}$

w_i satisfies $(L+G)w = \mathbf{1}_N$

Differentiating *V*

$$\dot{V} = e_\omega^T P(L+G)(\dot{\omega}) = e_\omega^T P(L+G)(u_\omega) \Longrightarrow \dot{V} = \frac{-c_\omega}{2} e_\omega^T (P(L+G) + (L+G)^T P) e_\omega$$

$$\Longrightarrow \quad \dot{V} < 0$$

Theorem of Zhihua Qu

Coop ctrl Lyapunov equation $Q = P(L+G) + (L+G)^T P > 0$

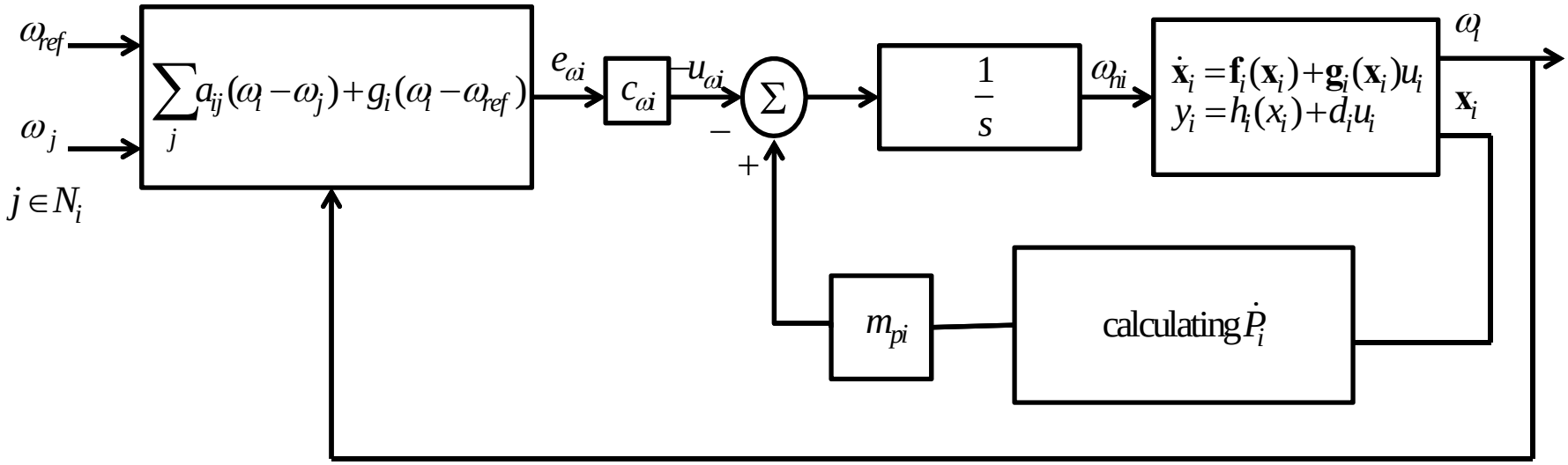
Qu, Z., (2009).*Cooperative control of dynamical systems: Applications to autonomous vehicles*. New York: Springer-Verlag.

Internal Dynamics is stable for AC microgrid DG

1. Secondary Frequency Control

Restores Frequency Synchronization after islanding

$$\dot{\omega}_i = \dot{\omega}_{ni} - m_{Pi}\dot{P}_i \equiv u_{\omega i}$$



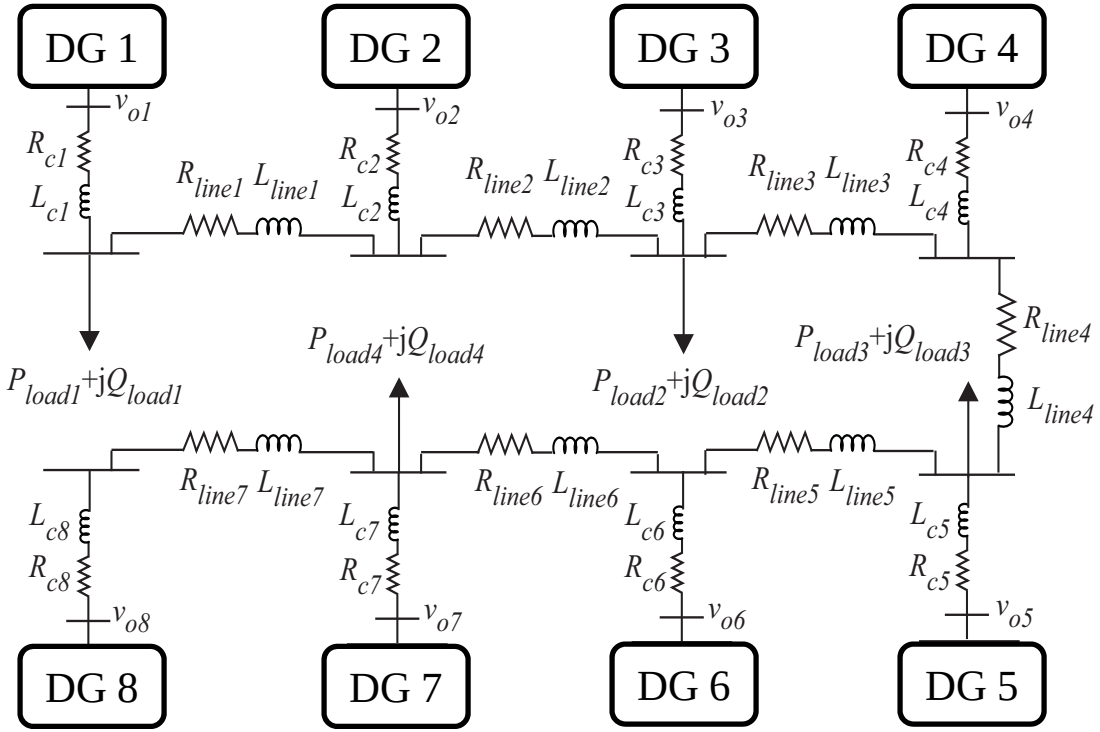
Feedback Linearization Inner Loop

Distributed Cooperative Tracker

1. Secondary frequency control

Simulation Example

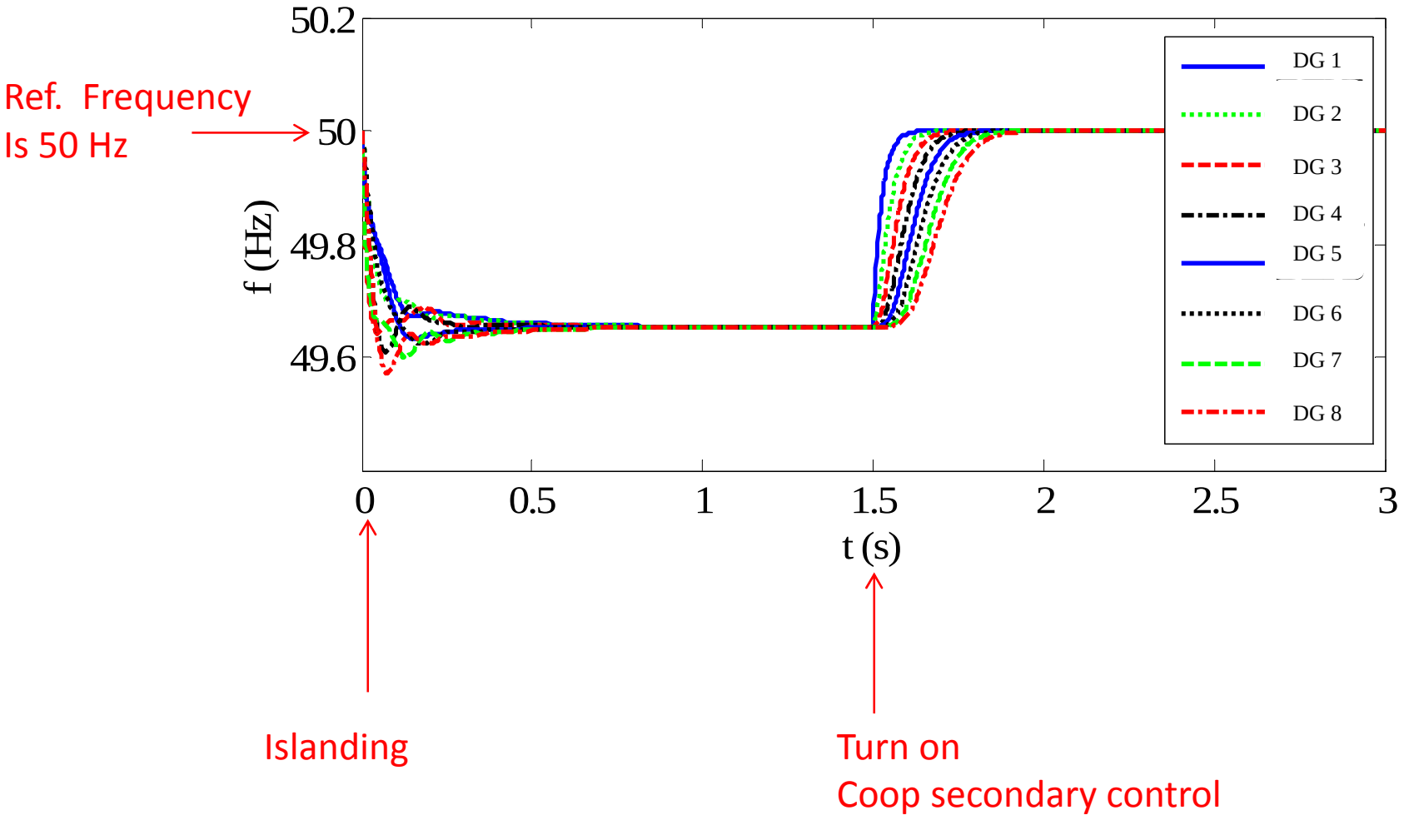
Physical Microgrid Network



Cyber communication network- sparse



1. Secondary frequency control



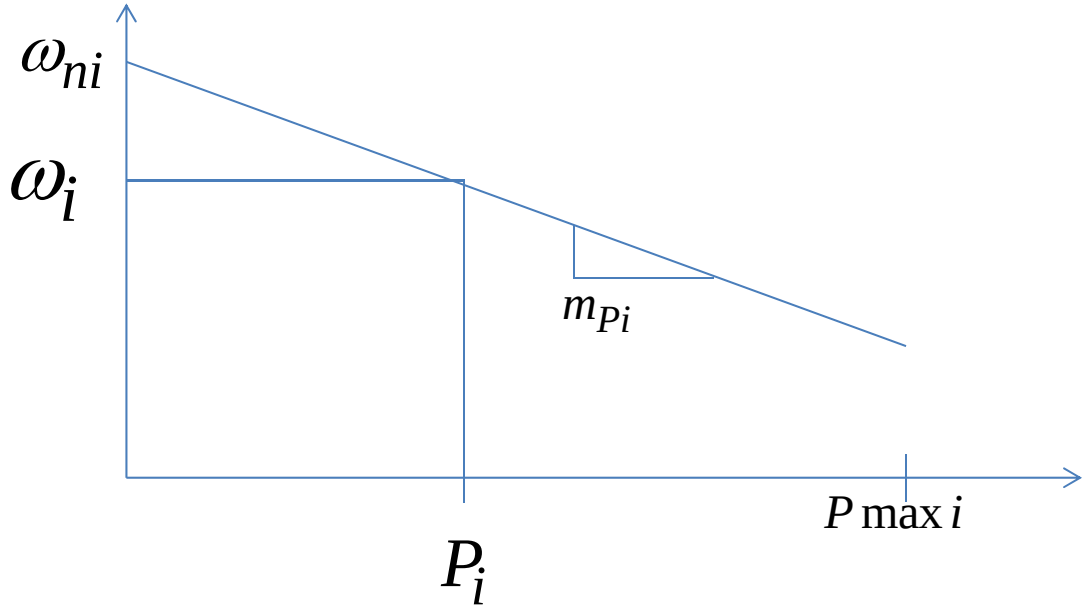



Shi nian su mu
Bai nian su ren
十年树木，百年树人
Keshi-
Wu nian su shuesheng
www.chinahighlights.com

New Secondary Control Input for Frequency Synchronization AND Balanced Power Sharing

$$\omega_i = \omega_{ni} - m_{Pi} P_i$$

Secondary Control input



Change ω_{ni}

To synchronize ω_i

And enforce balanced power sharing $\dfrac{P_1}{P_{\max 1}} = \cdots = \dfrac{P_N}{P_{\max N}}$

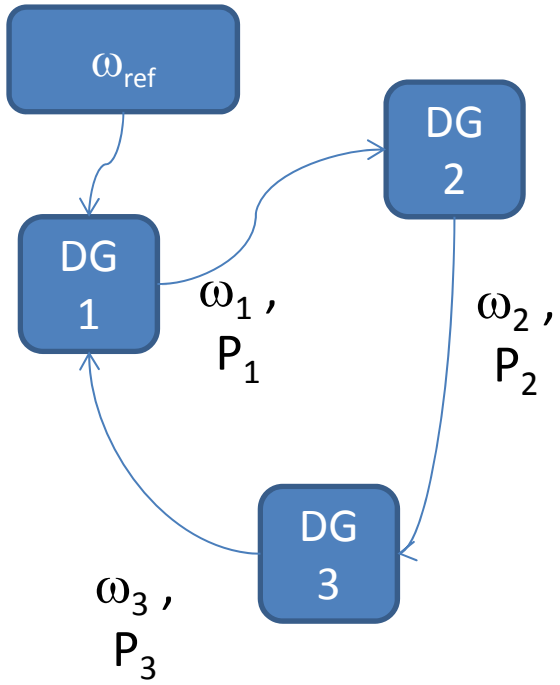
Secondary frequency and power control

Differentiating the frequency droop characteristic $\omega_i = \omega_{ni} - m_{Pi}P_i$ yields:

$$\dot{\omega}_{ni} = \dot{\omega}_i + m_{Pi}\dot{P}_i = u_i$$

For *N* DGs:

$$\begin{cases} \dot{\omega}_1 + m_{P1}\dot{P}_1 = u_1 \\ \dot{\omega}_2 + m_{P2}\dot{P}_2 = u_2 \\ \vdots \\ \dot{\omega}_N + m_{PN}\dot{P}_N = u_N \end{cases}$$



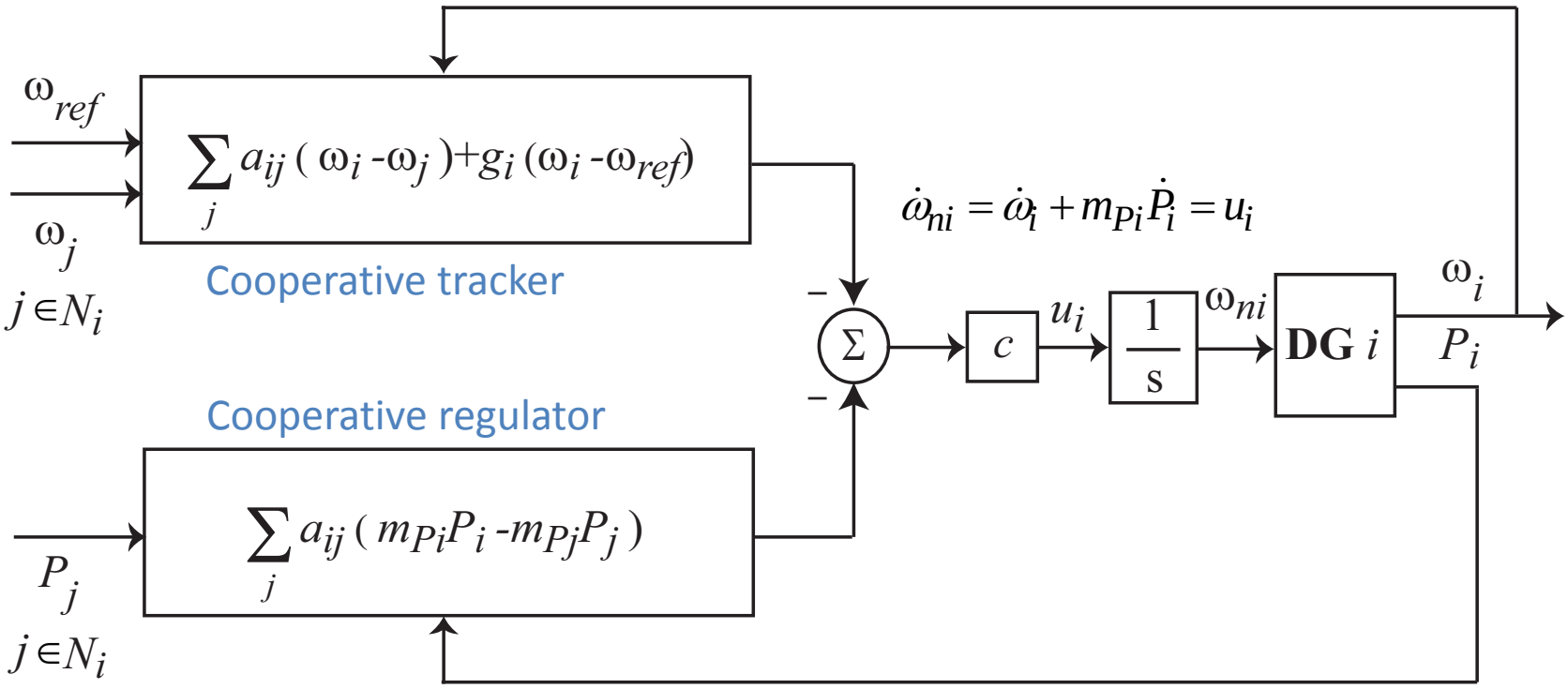
The auxiliary controls are chosen based on each DG's own information, and the information of its neighbors in the communication digraph

Local neighborhood tracking errors

$$u_i = -c\left(\sum_{j\in N_i} a_{ij}(\omega_i - \omega_j) + g_i(\omega_i - \omega_{ref}) + \sum_{j\in N_i} a_{ij}(m_{Pi}P_i - m_{Pj}P_j)\right)$$

TWO CONTROL OBJECTIVES WITH ONE CONTROL INPUT

Secondary frequency and power control



TWO CONTROL OBJECTIVES WITH ONE CONTROL INPUT

Secondary frequency and power control

The local neighborhood tracking error control

$$u_i = -c(\sum_{j\in N_i} a_{ij}(\omega_i - \omega_j) + g_i(\omega_i - \omega_{ref}) + \sum_{j\in N_i} a_{ij}(m_{Pi}P_i - m_{Pj}P_j))$$

Guarantees that $-c((L+G)(\omega - \underline{\omega}_{ref}) + Lm_P P) = 0.$

This does not guarantee synchronization of freq. and power separately

---

There is another relation between power and frequency in the microgrid DGs

Write the output active power as $$P_i = \frac{|v_{oi}||v_{bi}|}{X_{ci}}\sin(\delta_i) \equiv h_i \sin(\delta_i),$$

So that approximately $$\dot{P}_i = h_i(\omega_i - \omega_{ref}),$$

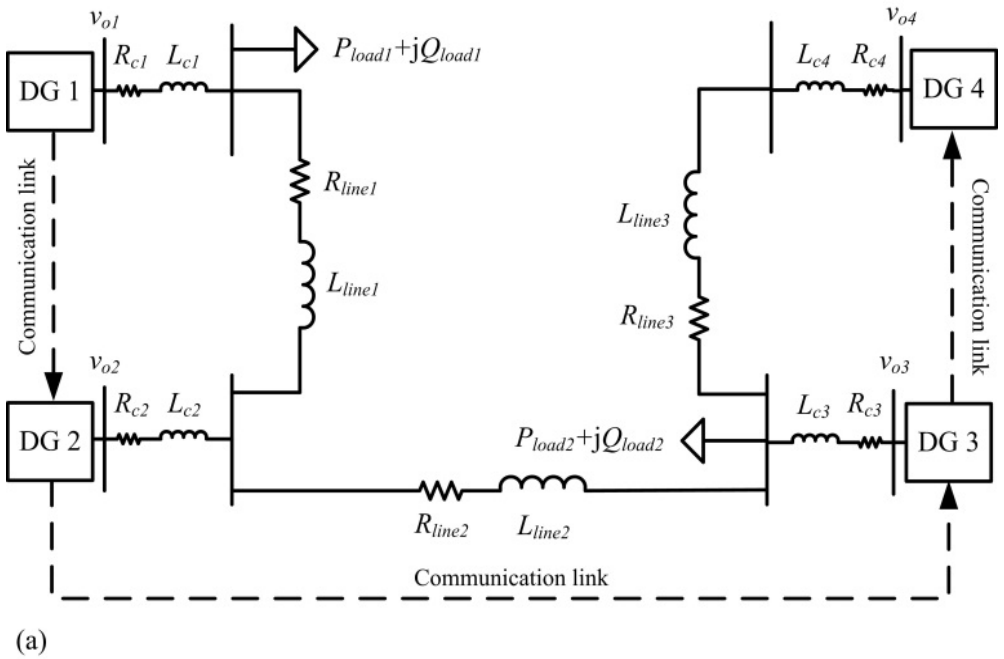
In global form $$\dot{P} = h(\omega - \underline{\omega}_{ref}),$$

Therefore at steady state all frequencies synchronize to the reference frequency

So that $$G(\omega - \underline{\omega}_{ref}) + Lm_P P = 0$$

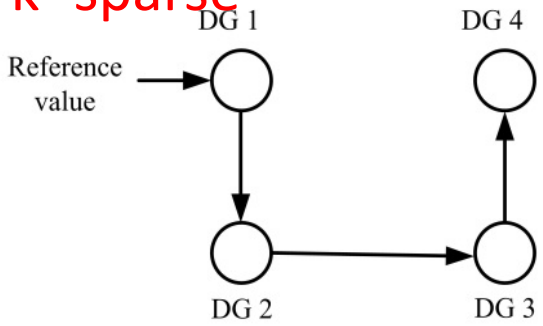
Simulation results

Physical Microgrid Network



(a)

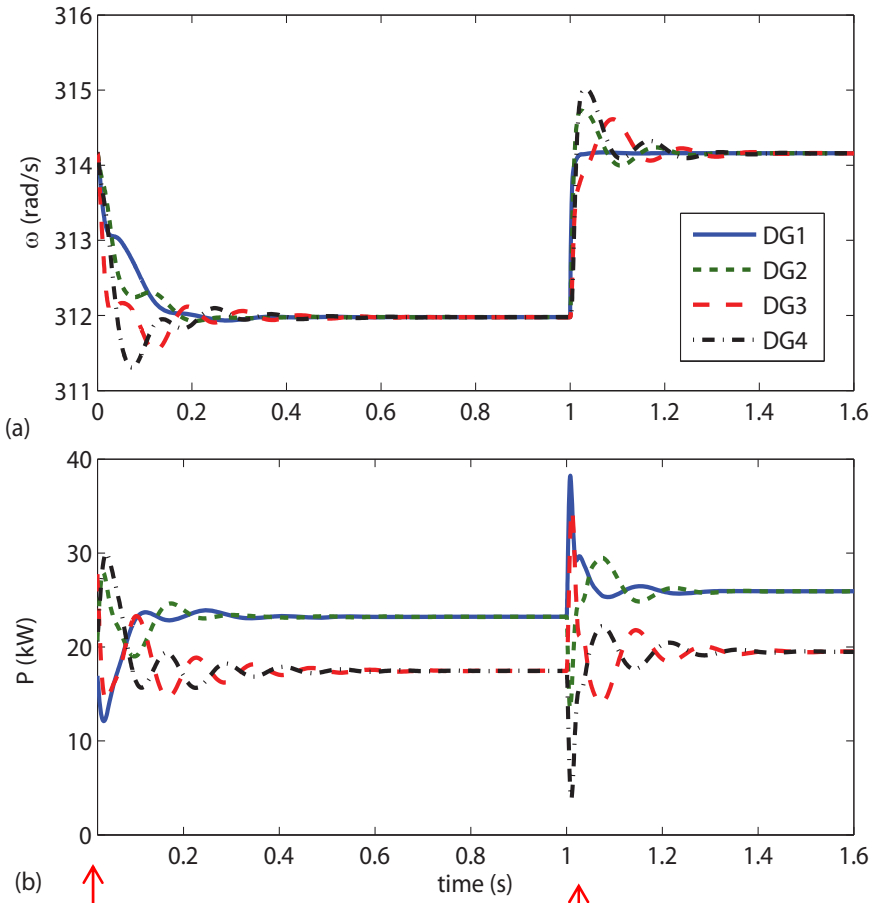
Cyber communication network- sparse



(b)

Simulation results

Ref. Frequency
Is 50 Hz



Islanding

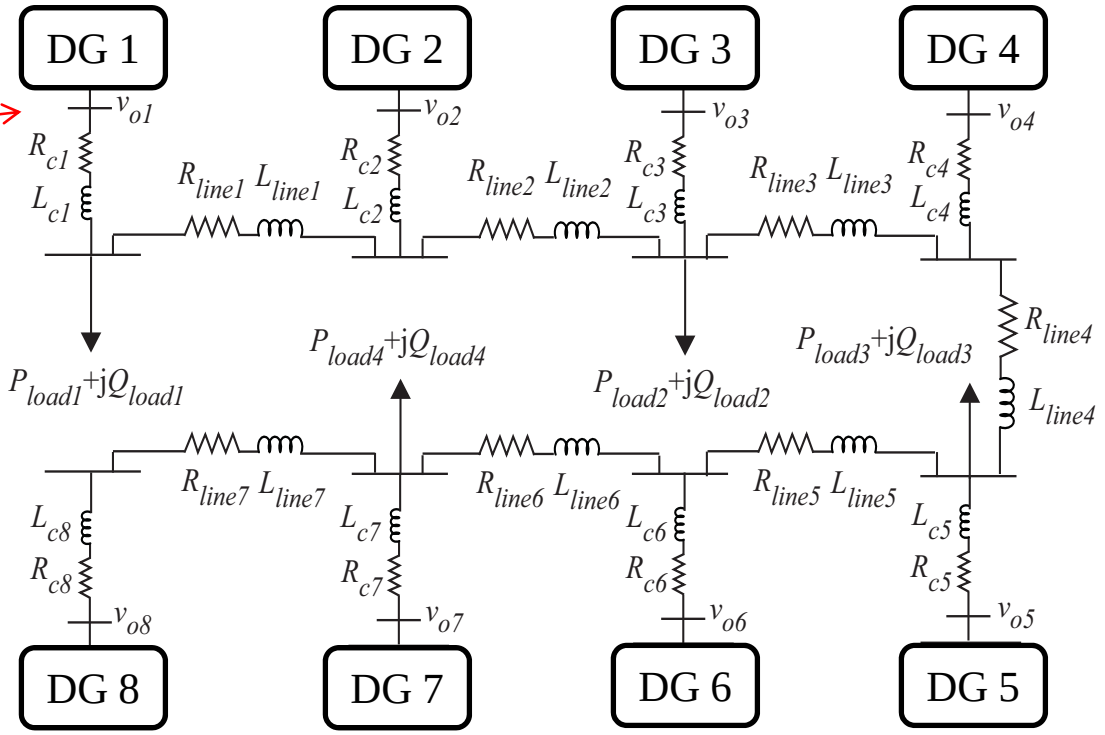
Turn on
Coop secondary control



2. Secondary Voltage Control Microgrid of Interconnected DG

Work of Ali Bidram
With Dr. A. Davoudi

Synchronize per-unit voltages



1. Frequency synchronization $y_i = \omega_i = \omega_{ni} - m_{Pi} P_i$

2. Voltage synchronization (per unit) $E \equiv v_{mag} = V_n - n_Q Q$

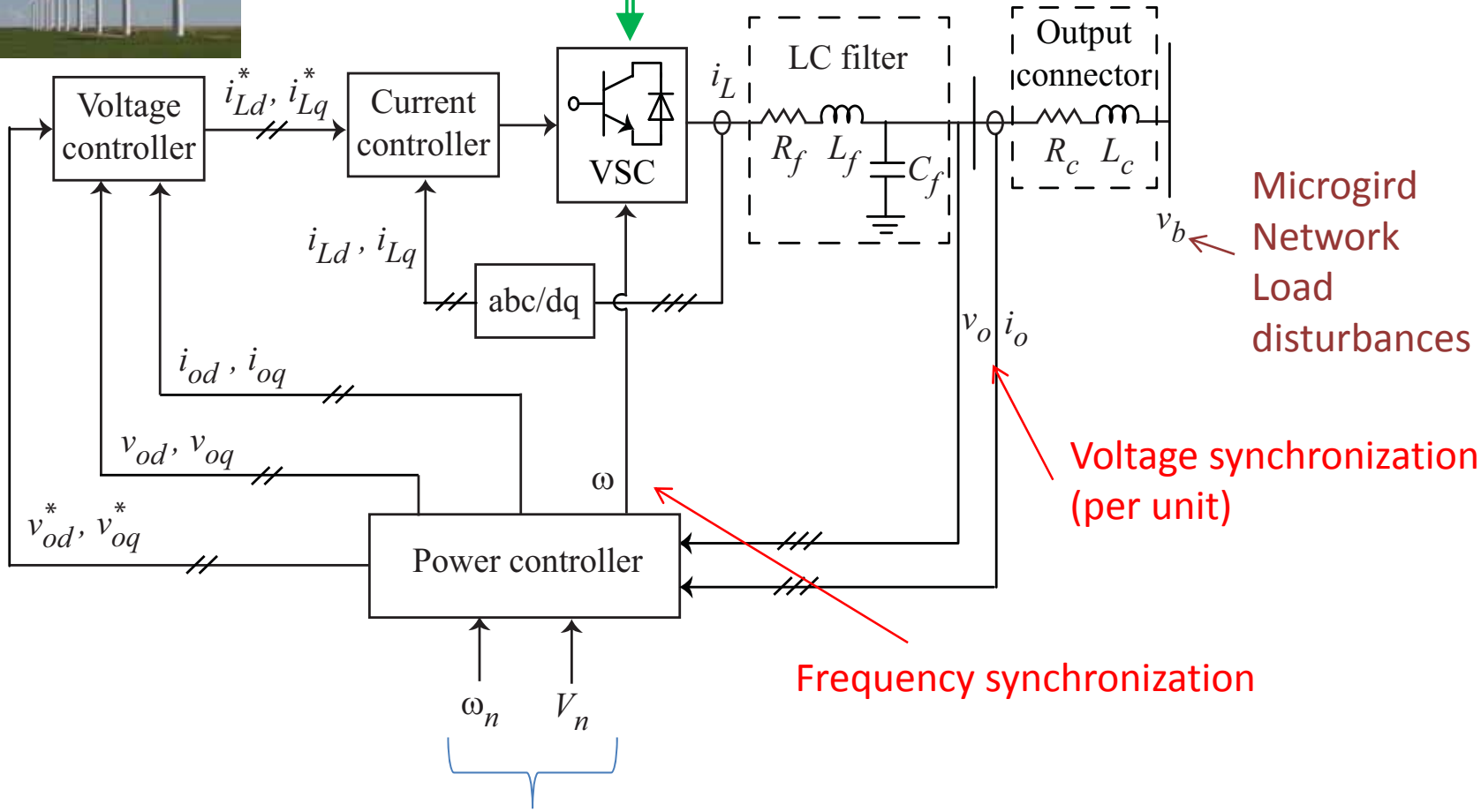
$$v_{o,magi} = \sqrt{v_{odi}^{\,2} + v_{oqi}^{\,2}}$$

2. Secondary Voltage control
Secondary Control Synchronization Objectives



Renewable DER
Provides DC voltage

VSC- Voltage source converter
Power electronics- DC to AC



Microgird
Network
Load
disturbances

Voltage synchronization
(per unit)

Frequency synchronization

Secondary Control Inputs
Change Droop control parameters to get synchronization

2. Secondary Voltage Control

Secondary Voltage control objective

The per unit voltage of each DG synchronizes to a command nominal value

Voltage synchronization (per unit)

$$v_{o,magi} = \sqrt{v_{odi}^{\,2} + v_{oqi}^{\,2}}$$

Droop Control

$$\begin{cases} \omega = \omega_n - m_P P \\ E \equiv v_{mag} = V_n - n_Q Q \end{cases}$$

Use this for frequency synchronization

Use this for voltage synchronization

2. Secondary Voltage Control

If $y_i = v_{odl}$ there is no direct relationship between the output and input $u_i = V_{ni}$. $d_i = 0$

Must use Lie derivatives

Input-output feedback linearization for a heterogeneous nonlinear agents

$$\begin{cases} \dot{\mathbf{x}}_i = \mathbf{f}_i(\mathbf{x}_i) + \mathbf{k}_i(\mathbf{x}_i)\mathbf{D}_i + \mathbf{g}_i(\mathbf{x}_i)u_i \\ y_i = h_i(\mathbf{x}_i) \end{cases} \qquad \mathbf{F}_i(\mathbf{x}_i) \equiv \mathbf{f}_i(\mathbf{x}_i) + \mathbf{k}_i(\mathbf{x}_i)\mathbf{D}_i$$

$$y_i^{(r)} = L_{\mathbf{F}_i}{}^r h_i + L_{\mathbf{g}_i} L_{\mathbf{F}_i}{}^{r-1} h_i u_i$$

$$v_i \equiv L_{\mathbf{F}_i}{}^r h_i + L_{\mathbf{g}_i} L_{\mathbf{F}_i}{}^{r-1} h_i u_i \Rightarrow u_i = (L_{\mathbf{g}_i} L_{\mathbf{F}_i}{}^{r-1} h_i)^{-1}(-L_{\mathbf{F}_i}{}^r h_i + v_i)$$

$$y_i^{(r)} = v_i, \forall i \Rightarrow \begin{cases} \dot{y}_i \equiv y_{i,1} \\ \dot{y}_{i,1} \equiv y_{i,2} \\ \vdots \\ \dot{y}_{i,r-1} = v_i \end{cases}, \forall i \Rightarrow \dot{\mathcal{Y}}_i = \mathbf{A}\mathcal{Y}_i + \mathbf{B}v_i, \forall i,$$

Internal Dynamics $\dot{\mu}_i = W_i(\mathcal{Y}_i, \mu_i), \forall i$

Assume relative degree *r* is the same for all agents

Zero dynamics can be different, but assume they are stable

2. Secondary voltage control

DG Agent Dynamics

$$\dot{\mathcal{Y}}_i = \mathbf{A}\mathcal{Y}_i + \mathbf{B}v_i, \forall i, \qquad \mathcal{Y}_i = [y_i \quad y_{i,1} \quad \cdots \quad y_{i,r-1}]^T$$

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}_{r\times r} \qquad \mathbf{B} = [0,0,\ldots,1]^T_{r\times 1}$$

Internal dynamics $\dot{\mu}_i = W_i(\mathcal{Y}_i, \mu_i), \forall i$

Leader node dynamics

$$\begin{cases} \dot{\mathbf{x}}_0 = \mathbf{f}_0(\mathbf{x}_0) \\ y_0 = h_0(\mathbf{x}_0) \end{cases} \Longrightarrow \dot{\mathbf{Y}}_0 = \mathbf{A}\mathbf{Y}_0 + \mathbf{B}y_0^{(r)}, \quad \mathbf{Y}_0 = [y_0 \quad \dot{y}_0 \quad \cdots \quad y_0^{(r-1)}]^T$$

The synchronization problem is to find a distributed v_i such that $\mathcal{Y}_i \to \mathbf{Y}_0, \forall i.$

Assumption. The vector $\underline{\mathbf{y}}_0^{(r)} = \mathbf{1}_N y_0^{(r)}, \forall r$ is bounded so that $\left\|\underline{\mathbf{y}}_0^{(r)}\right\| \le Y_M^r$, with a finite but generally unknown bound.

2. Secondary voltage control

Theorem. Let the digraph of the multi-agent system have a spanning tree and the pinning gain be nonzero for at least one root node.

Let all agents have stable zero dynamics $\dot{\mu}_i = W_i(0, \mu_i), \forall i$

Let the auxiliary control be chosen as

$$v_i = -cK\mathbf{e}_i \qquad \mathbf{e}_i = \sum_{j \in N_i} a_{ij}(\mathcal{Y}_i - \mathcal{Y}_j) + g_i(\mathcal{Y}_i - \mathbf{Y}_0)$$

where $c \in R$ is the coupling gain, and $K \in R^{1\times r}$ is the feedback control gain.

Then, $\mathcal{Y}_i$ are cooperative UUB with respect to $\mathbf{Y}_0$ and all nodes synchronize to $\mathbf{Y}_0$ if $K \in R^{1\times r}$ is chosen as

$$K = R^{-1}\mathbf{B}^T P_1, \qquad \mathbf{A}^T P_1 + P_1\mathbf{A} + Q - P_1\mathbf{B}R^{-1}\mathbf{B}^T P_1 = 0.$$

and $c \geq \dfrac{1}{2\lambda_{\min}}, \quad \lambda_{\min} = \min_{i\in\mathcal{N}} Re(\lambda_i)$ λ_i is the first eigenvalue of $L + G$

Zhang, H., Lewis, F. L., & Das, A. (2011).Optimal design for synchronization of cooperative systems: State feedback, observer, and output feedback. *IEEE Transactions on Automatic Control*, *56*(8), 1948–1952.

2. Secondary voltage control

Lyapunov function

$$V = \frac{1}{2}\boldsymbol{\delta}^T P_2 \boldsymbol{\delta}, \quad P_2 = P_2^T,\ P_2 > 0,$$

Differentiating $\dot{V} = \boldsymbol{\delta}^T P_2 \dot{\boldsymbol{\delta}} = \boldsymbol{\delta}^T P_2 \left(\dot{\mathcal{Y}} - \dot{\mathcal{Y}}_0\right) = \boldsymbol{\delta}^T P_2 \left((I_N \otimes \mathbf{A})\boldsymbol{\delta} + (I_N \otimes \mathbf{B})(v - \underline{\mathbf{y}}_0^{(r)})\right)$

$$\dot{V} = \boldsymbol{\delta}^T P_2 \left(I_N \otimes \mathbf{A} - c(L+G) \otimes \mathbf{B}K\right)\boldsymbol{\delta} - \boldsymbol{\delta}^T P_2 (I_N \otimes \mathbf{B})\underline{\mathbf{y}}_0^{(r)}$$

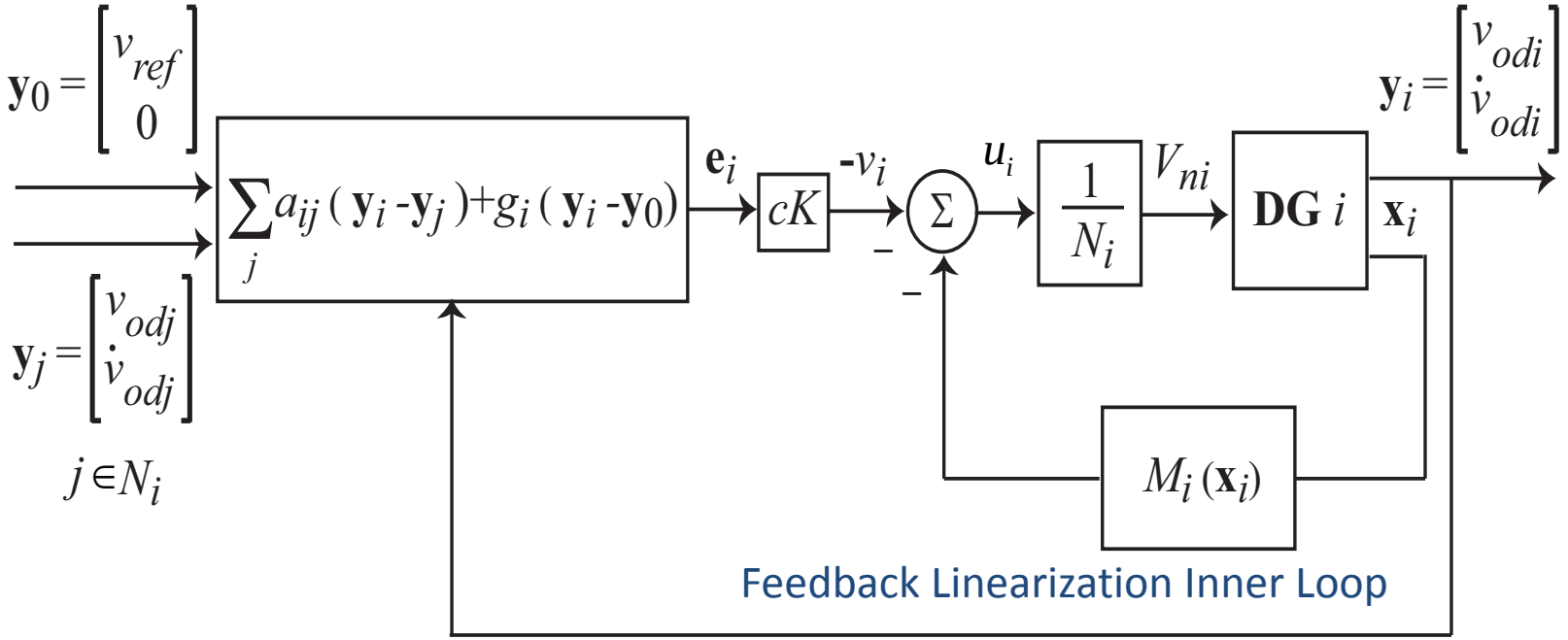
$$\equiv \boldsymbol{\delta}^T P_2 \mathbf{H}\boldsymbol{\delta} - \boldsymbol{\delta}^T P_2 (I_N \otimes \mathbf{B})\underline{\mathbf{y}}_0^{(r)}.$$

Lemma 1. Let $(\mathbf{A},\mathbf{B})$ be stabilizable. Let the digraph have a spanning tree and $g_i \neq 0$ for at least one root node. Let λ_i be the eigenvalues of $L+G$. The matrix $\mathbf{H} = I_N \otimes \mathbf{A} - c(L+G) \otimes \mathbf{B}K$ is Hurwitz if and only if all the matrices $\mathbf{A} - c\lambda_i \mathbf{B}K, \ \forall i \in \mathcal{N}$ are Hurwitz. [Fax and Murray 2004]

Lemma 2. Let $(\mathbf{A},\mathbf{B})$ be stabilizable and matrices $Q = Q^T$ and $R = R^T$ be positive definite. Let feedback gain K be chosen as $K = R^{-1}\mathbf{B}^T P_1$, where P_1 is the unique positive definite solution of the control algebraic Riccati equation $\mathbf{A}^T P_1 + P_1 \mathbf{A} + Q - P_1 \mathbf{B} R^{-1} \mathbf{B}^T P_1 = 0.$ Then, all the matrices $\mathbf{A} - c\lambda_i \mathbf{B}K$ are Hurwitz if $c \geq \frac{1}{2\lambda_{\min}}$, where $\lambda_{\min} = \min_{i \in \mathcal{N}} Re(\lambda_i)$. [Zhang and Lewis 2011]

Secondary voltage control

Synchronizes Output voltages after Islanding



$$y_i^{(2)} = L_{\mathbf{F}_i}{}^2 h_i + L_{\mathbf{g}_i} L_{\mathbf{F}_i}{}^1 h_i u_i$$

$$v_i = L_{\mathbf{F}_i}{}^2 h_i + L_{\mathbf{g}_i} L_{\mathbf{F}_i}{}^1 h_i u_i \quad \Rightarrow$$

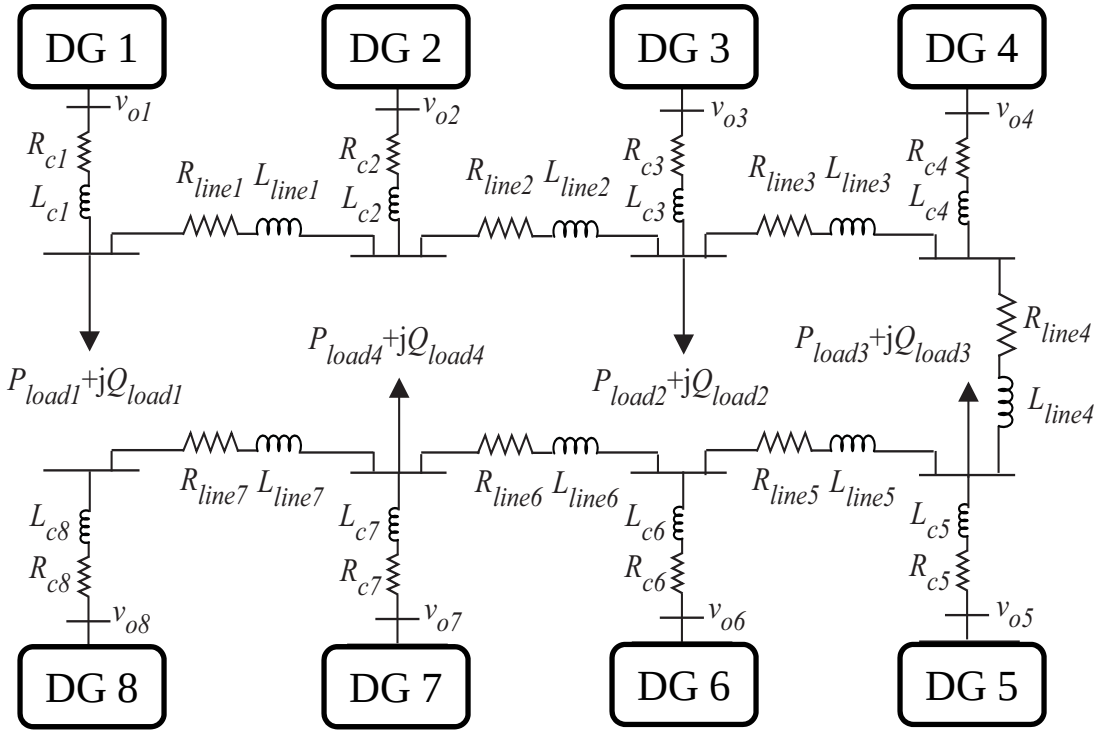
$$u_i = (L_{\mathbf{g}_i} L_{\mathbf{F}_i}{}^1 h_i)^{-1}(-L_{\mathbf{F}_i}{}^2 h_i + v_i)$$

$$V_{ni} = \frac{v_i - M_i(\mathbf{x}_i)}{N_i}, \forall i.$$

2. Secondary voltage control

Simulation Example

Physical Microgrid Network



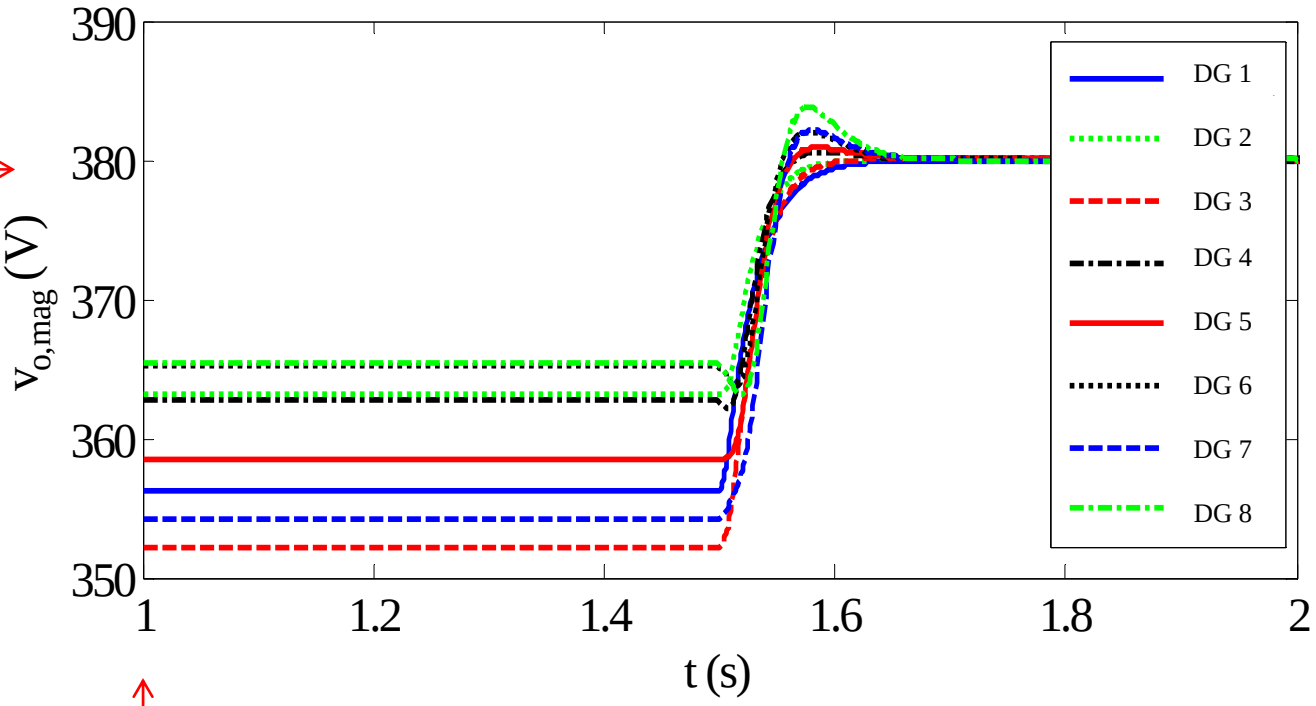
Cyber communication network- sparse



2. Secondary voltage control

Simulation results

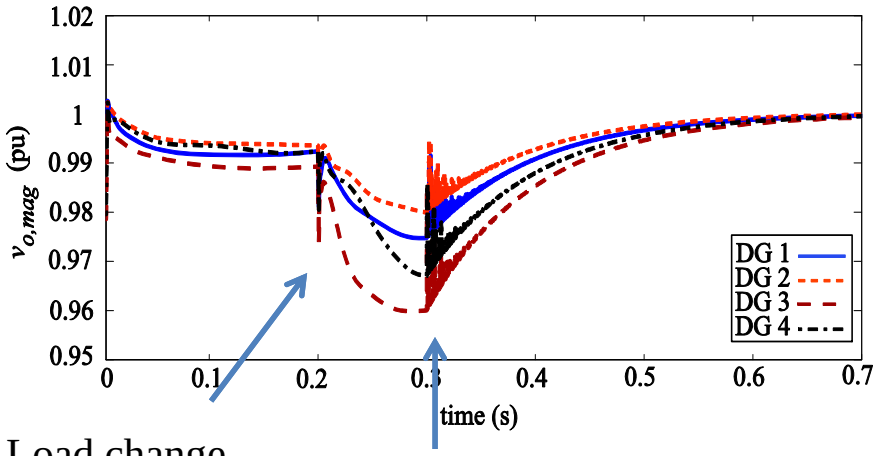
Ref. Per-unit Voltage Is 380 V



Islanding

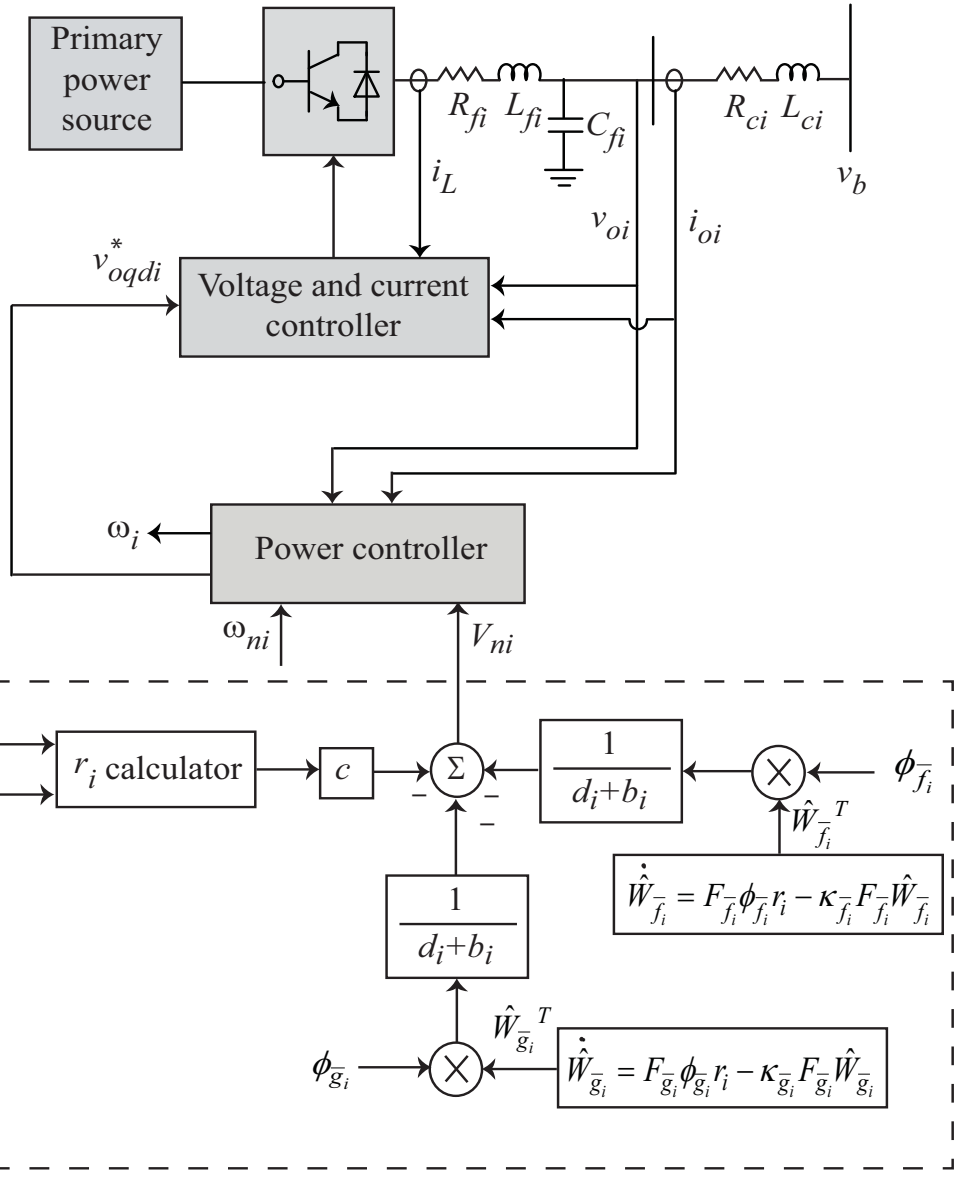
Turn on
Coop secondary control

Adaptive Voltage Control



Load change
Primary alone!

Secondary



$$u_i = (L_{\mathbf{g}_i} L_{\mathbf{F}_i}{}^1 h_i)^{-1}(-L_{\mathbf{F}_i}{}^2 h_i + v_i)$$

Using Neural Network to compensate for unknown nonlinear dynamics

$$M_i(x_i),\ N_i(x_i)$$

A Distributed Feedforward Approach to Cooperative Control of AC Microgrids

He Cai, *Member, IEEE*, Guoqiang Hu, *Member, IEEE*, Frank L. Lewis, *Life Fellow, IEEE*, and Ali Davoudi, *Senior Member, IEEE*

Voltage controlled VSI

1) VCVSIs: For $i = 1, \ldots, M$, the nonlinear dynamics of the i^{th} VCVSI can be written as

$$\dot{x}_{i1} = -a_{i1}x_{i1} + \omega_i x_{i2} + a_{i2}(u_{i1} - x_{i3}) \quad (1a)$$
$$\dot{x}_{i2} = -a_{i1}x_{i2} - \omega_i x_{i1} + a_{i2}(u_{i2} - x_{i4}) \quad (1b)$$
$$\dot{x}_{i3} = \omega_i x_{i4} + a_{i3}(x_{i1} - x_{i5}) \quad (1c)$$
$$\dot{x}_{i4} = -\omega_i x_{i3} + a_{i3}(x_{i2} - x_{i6}) \quad (1d)$$
$$\dot{x}_{i5} = -a_{i4}x_{i5} + \omega_i x_{i6} + a_{i5}(x_{i3} - v_{bdi}) \quad (1e)$$
$$\dot{x}_{i6} = -a_{i4}x_{i6} - \omega_i x_{i5} + a_{i5}(x_{i4} - v_{bqi}) \quad (1f)$$
$$\dot{x}_{i7} = \omega_{ci}(x_{i3}x_{i5} + x_{i4}x_{i6} - x_{i7}) \quad (1g)$$
$$\dot{x}_{i8} = \omega_{ci}(x_{i3}x_{i6} - x_{i4}x_{i5} - x_{i8}) \quad (1h)$$
$$y_{i1} = x_{i3} \quad (1i)$$
$$y_{i2} = x_{i4} \quad (1j)$$

As above

Current controlled VSI

2) CCVSIs: For $i = M+1, \ldots, N$, the nonlinear dynamics of the i^{th} CCVSI can be written as

$$\dot{x}_{i1} = -a_{i1}x_{i1} + \omega_i x_{i2} + a_{i2}(u_{i1} - v_{bdi}) \quad (2a)$$
$$\dot{x}_{i2} = -a_{i1}x_{i2} - \omega_i x_{i1} + a_{i2}(u_{i2} - v_{bqi}) \quad (2b)$$
$$y_{i1} = x_{i1} \quad (2c)$$
$$y_{i2} = x_{i2} \quad (2d)$$

Voltage Synchronization

VCVSI are second order after FB lin

Let $x_i = \text{col}(x_{i1}, x_{i2}, x_{i3}, x_{i4}, x_{i5}, x_{i6}, x_{i7}, x_{i8})$, $\bar{x}_i = \text{col}(\bar{x}_{i1}, \bar{x}_{i2})$ and $\tilde{x}_i = \text{col}(\tilde{x}_{i1}, \tilde{x}_{i2})$. Then we have

$$\dot{\bar{x}}_i = A\bar{x}_i + B(b_i u_{i1} + \alpha_{i1}(x_i, d_i)) \quad (9a)$$

$$\dot{\tilde{x}}_i = A\tilde{x}_i + B(b_i u_{i2} + \alpha_{i2}(x_i, d_i)) \quad (9b)$$

where $b_i = a_{i2}a_{i3}$,

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \; B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad (10)$$

CCVSI are first order after FB lin

Similarly, for the CCVSIs, letting $x_i = \text{col}(x_{i1}, x_{i2})$, $\bar{x}_i = x_{i1}$, $\tilde{x}_i = x_{i2}$ gives

$$\dot{\bar{x}}_i = \bar{A}\bar{x}_i + \bar{B}(b_i u_{i1} + \alpha_{i1}(x_i, d_i)) \quad (13a)$$

$$\dot{\tilde{x}}_i = \bar{A}\tilde{x}_i + \bar{B}(b_i u_{i2} + \alpha_{i2}(x_i, d_i)) \quad (13b)$$

where $b_i = a_{i2}$,

$$\bar{A} = 0, \; \bar{B} = 1 \quad (14a)$$

Heterogeneous MAS

Solution of Output regulator Equations for Chained Form Systems

Theorem 1: Let (X, U), where $X \in R^{n\times n_0}$ and $U \in R^{1\times n_0}$ be the solution pairs of the following regulator equations

$$XS = AX + BU,\ 0 = CX - F \tag{17}$$

where

$$S = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix} \in R^{n_0\times n_0}$$

$$A = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix} \in R^{n\times n},\ B = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} \in R^n$$

$$C = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \end{bmatrix} \in R^{1\times n}$$

$$F = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \end{bmatrix} \in R^{1\times n_0}. \tag{18}$$

Define the following matrix

$$M = \begin{pmatrix} I_{n_0} \\ 0_{(m-n_0)\times n_0} \end{pmatrix} \in R^{m\times n_0} \tag{19}$$

where $m > n$. Let m_k be the k^{th} row of M. Then,

$$X = \begin{bmatrix} m_1 \\ \vdots \\ m_n \end{bmatrix},\ U = \begin{bmatrix} 0 & m_n^1 & \cdots & m_n^{n_0-1} \end{bmatrix} \tag{20}$$

where m_n^j is the j^{th} element of m_n.

Jie Huang
Lu Liu

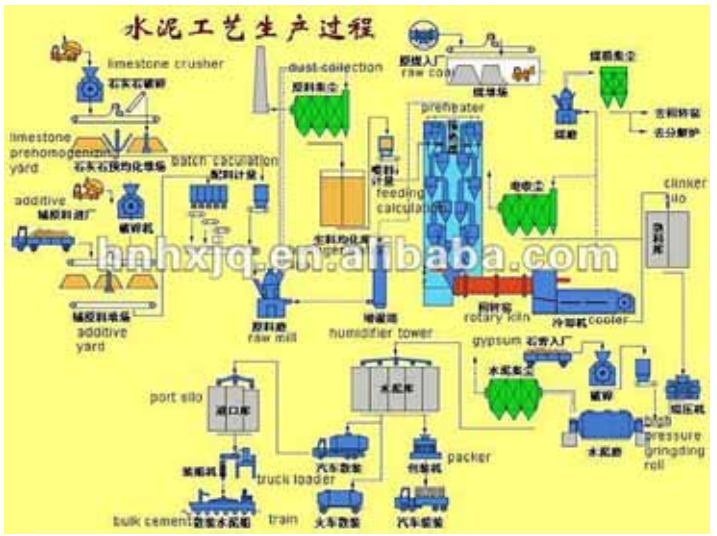
He Cai



Manufacturing as the Interactions of Multiple Agents

Each machine has it own dynamics and cost function
Neighboring machines influence each other most strongly
There are local optimization requirements as well as global necessities

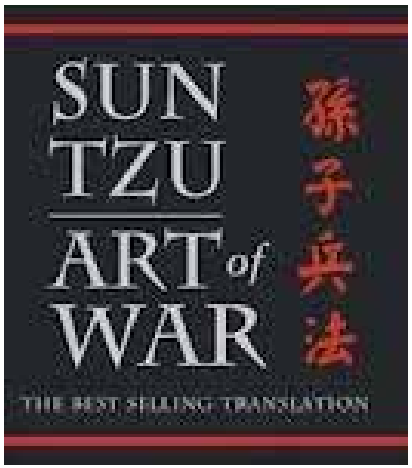


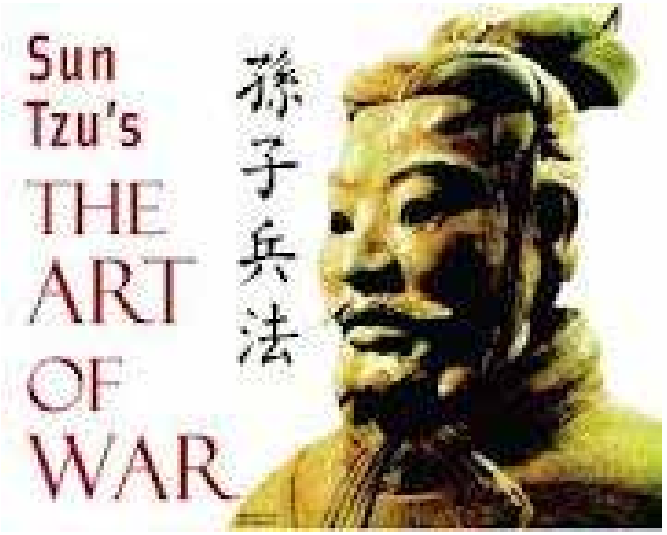






Games on Communication Graphs









孙**子兵法**

500 BC

Sun Tz bin fa

Intelligent Operational Control for Complex Industrial Processes

Jinliang Ding

Professor Chai Tianyou

State Key Laboratory of Synthetical Automation for Process Industries

Northeastern University

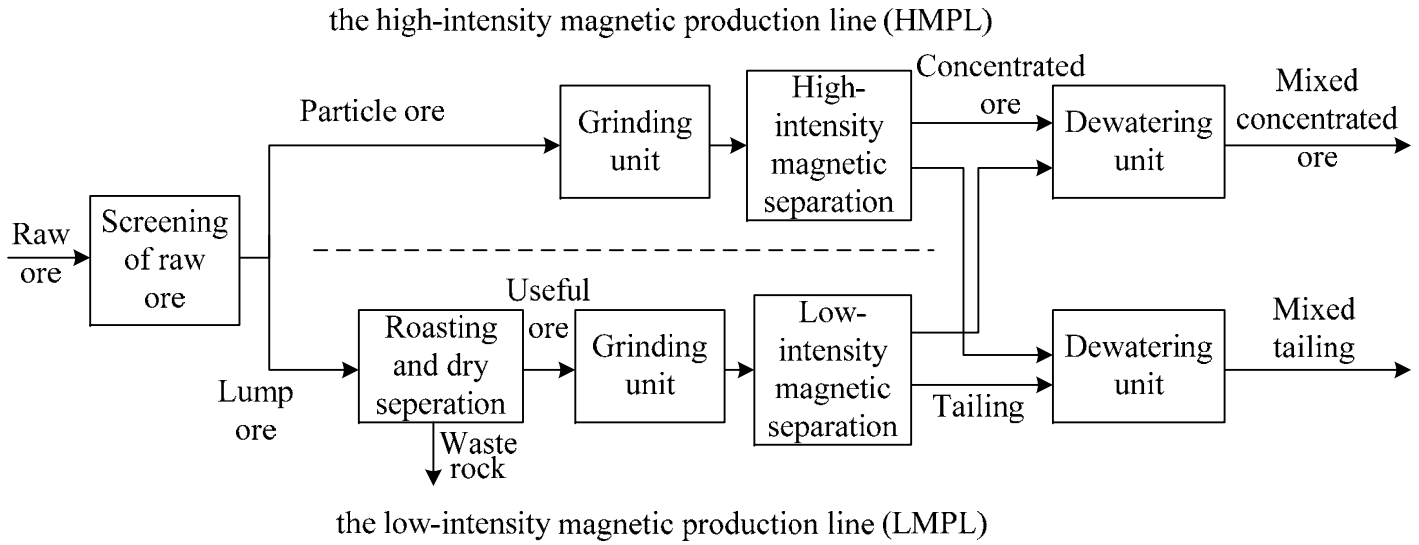
May 20, 2013

Jinliang Ding, H. Modares, Tianyou Chai, and F.L. Lewis, "Data-based Multi-objective Plant-wide Performance Optimization of Industrial Processes under Dynamic Environments," IEEE Trans. Industrial Informatics, to appear, 2016.

Production line for mineral processing plant







Mineral Processing Plant in Gansu China

