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EE 4343/5329 - Control System Design Project
LECTURE 7

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RATE LOOP/POSITION TRACKER DESIGN

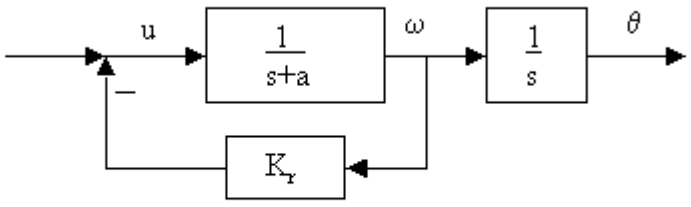
When there is more than one feedback loop, one can often use several root locus designs. An example is the design of a position tracking system by first closing an INNER RATE LOOP.

Example- Motor Control

A simple model of a motor is $H(s) = \frac{1}{s+a}$, which has an impulse response of e^{-at} u$_{-1}$(t). The input u(t) may be interpreted as a voltage, and the output as angular velocity $w(t) = \dot{q}(t)$. It is desired to design a controller that makes the angle θ(t) follow a prescribed angle command r(t). An application is a satellite tracking system which should point to the angle of the satellite, or a telescope which should point towards a specific star as the earth revolves.

Inner Rate Loop Design

To speed up the response for tracking, we first close an inner loop using rate feedback as shown, where the rate gain is K_r.

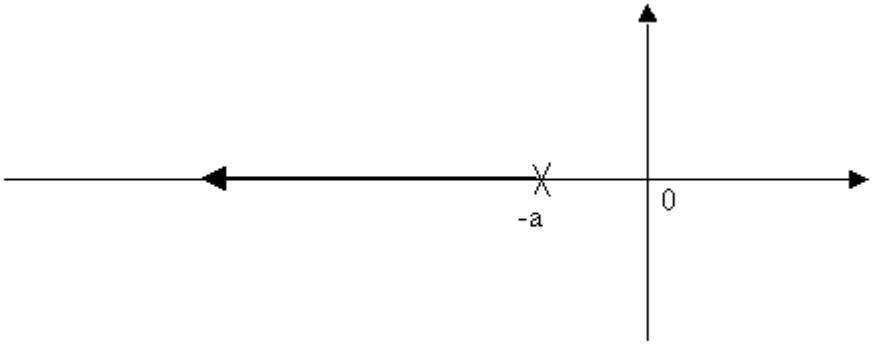


The closed-loop transfer function to ω(t) with rate feedback is

$$T_r(s) = \frac{\dfrac{1}{s+a}}{1 + K_r \dfrac{1}{s+a}} = \frac{1}{s + (a + K_r)}$$

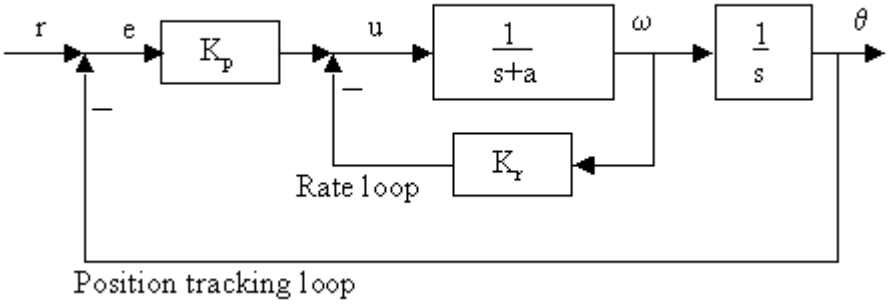
so that the closed-loop pole is now at s= -(a+K_r). This speeds up the response to $e^{-(a+K_r)t}u_{-1}(t)$.

The root locus as the rate feedback gain K_r is varied is shown in the figure. It is clear that the function of the rate feedback loop is to speed up the system by moving the pole originally at s= -a to the left.



Outer Position Loop Design

Once the system has been sped up using rate feedback, one may close the position loop using position gain K_p.



The unity-gain outer loop reveals a tracker topology. Since there is an integrator in the feedforward loop, the system is of Type 1 and will exhibit zero steady-state error to a unit step angle command r(t).

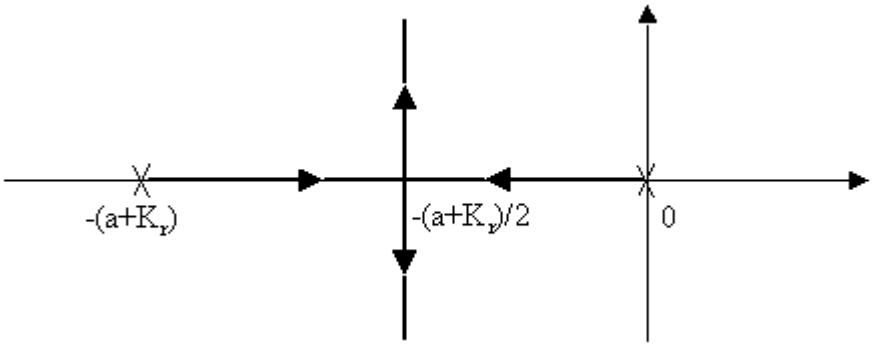
The closed-loop transfer function is

$$T(s) = \frac{K_p T_r(s)\dfrac{1}{s}}{1 + K_p T_r(s)\dfrac{1}{s}}$$

so that the open-loop gain for plotting a root locus versus Kp is

$$G(s) = T_r(s)/s = \frac{1}{[s + (a + K_r)]s}.$$

The relative degree of G(s) is m=2 and the centroid for the asymptotes is at $c = -(a + K_r)/2$. The root locus versus position gain K_p for a fixed rate gain K_r is shown.



It is clear that by adjusting first K_r then K_p one can obtain any damping ratio and settling time one desires.

The closed-loop transfer function simplifies to

$$T(s) = \frac{K_p}{s^2 + (a + K_r)s + K_p}.$$

Comparing the denominator to $s^2 + 2as + w_n^2 = s^2 + 2zw_n s + w_n^2$ one may select the rate and position gains to obtain a desired damping ratio ζ and settling time $t_s = 5t$, $t = 1/a$.