# EE 4314- CONTROL SYSTEMS
# PID Compensators

**Updated: Tuesday, September 26, 2017**

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## PID COMPENSATORS

When the characteristics of a plant are not suitable, they can be changed by adding a compensator in the control system. Some basic sorts of compensators useful in feedback control design are discussed, including Proportional-Integral-Derivative (PID), lead, and lag.

## Compensator Transfer Functions

A tracking control system is given in the figure. The tracker topology is often used since it has a unity-gain outer loop that often aides in analysis and understanding. The plant to be controlled is H(s) and the compensator is kK(s), with k a constant gain. The function of this controller is to make the output follow the reference input r(t) by keeping small the tracking error e(t)=r(t)-y(t). The disturbance is denoted as d(t).

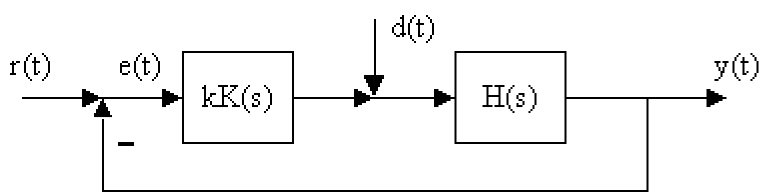



**Figure 1. Closed-Loop Tracking System with Feedforward Compensator**

The closed-loop transfer function is

$$T(s) = \frac{Y(s)}{R(s)} = \frac{kK(s)H(s)}{1 + kK(s)H(s)}.$$

The denominator

$$\Delta(s) = 1 + kK(s)H(s) \equiv 1 + kG(s)$$

is a polynomial fraction whose numerator is the closed-loop characteristic polynomial. $G(s) = K(s)H(s)$ is the open-loop gain.

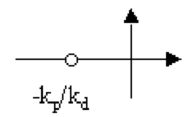
In guaranteeing stability and performance and shaping the closed-loop response, it is important to select a suitable compensator *kK(s)*. Several common sorts of compensators are given below.

**Proportional (P)**

$$kK(s) = k, \quad const$$

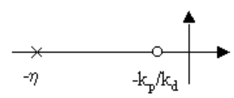
**Proportional-plus-Derivative (PD)**

$$kK(s) = k_d s + k_p = k_d (s + {}^{k_p}\!/_{k_d})$$



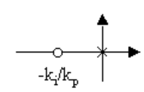

**Practical Proportional-plus-Derivative (PD)**

$$kK(s) = k_d \frac{s + {}^{k_p}\!/_{k_d}}{s + \eta}$$, where η is a large filtering pole



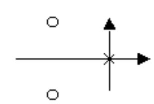

**Proportional-plus-Integral (PI)**

$$kK(s) = k_p + \frac{k_i}{s} = \frac{k_p s + k_i}{s} = k_p \frac{s + {}^{k_i}\!/_{k_p}}{s}$$



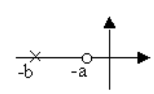

**PID**

$$kK(s) = k_d s + k_p + \frac{k_i}{s} = k_d \frac{s^2 + \frac{k_p}{k_d} s + \frac{k_i}{k_d}}{s}$$



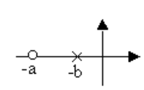
**Lead**

$$kK(s) = k \frac{s + a}{s + b}, \quad a < b$$




**Lag**

$$kK(s) = k \frac{s + a}{s + b}, \quad a > b$$




In practice, when using PD feedback one must include a filtering pole to smooth out noise in the differentiation process. Note that the filtering pole η changes the PD compensator for high frequencies, but the low frequency behavior is nearly the same as for the pure PD compensator.

## Ziegler-Nichols Tuning of PID Compensator

Consider the closed-loop system in Fig. 1 with a PID compensator kK(s). Three parameters must be adjusted in the PID controller, $k_d$, $k_p$, and $k_i$. Ziegler and Nichols provided a technique for selecting the PID gains that works for a large class of industrial systems.

First, set the derivative and integral gains equal to zero, closing only the P loop. Increase the P gain $k_p$ until the system just oscillates (so that the closed-loop poles are on the $j\omega$-axis). Determine the P gain $K_m$ at which this occurs, and the oscillation frequency $\omega_m$ at this point. Then, good values of PID gains are computed according to

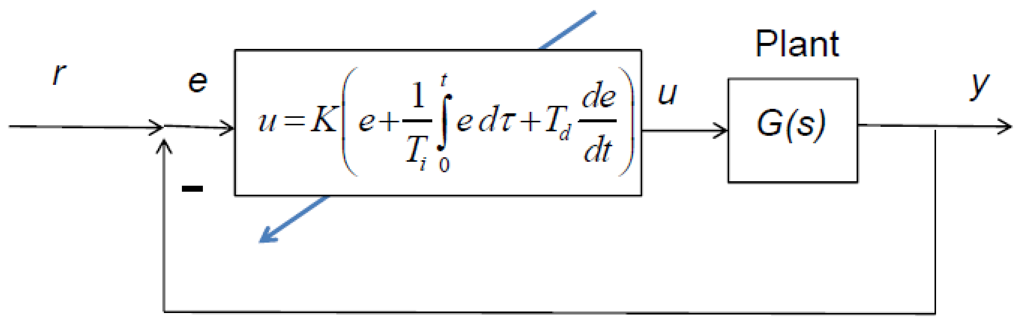
$$k_p = 0.6K_m, \quad k_d = \frac{k_p \pi}{4\omega_m}, \quad k_i = \frac{k_p \omega_m}{\pi}.$$

This technique has been found over the years to give very good results for many systems. It relies on the fact that many industrial processes effectively have a relative degree of three due to a dominant low frequency mode, so that as the P gain increases, the closed-loop poles cross the $j\omega$-axis and migrate into the right-half plane. We shall cover this when we talk about Root Locus design

# Autotuning

## 1. Tuning of Controller

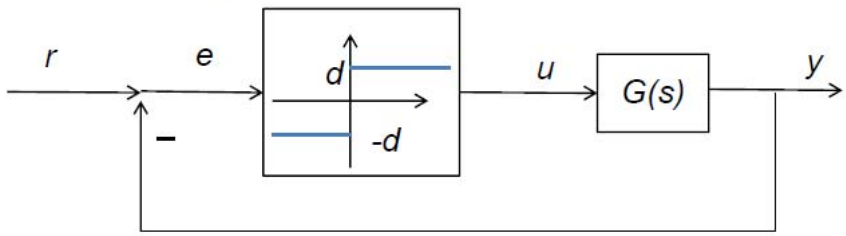
Online tuning of PID gains in real time for unknown plants




Tune control parameters *K*, $T_i$, $T_d$ online using real time measurements from Plant

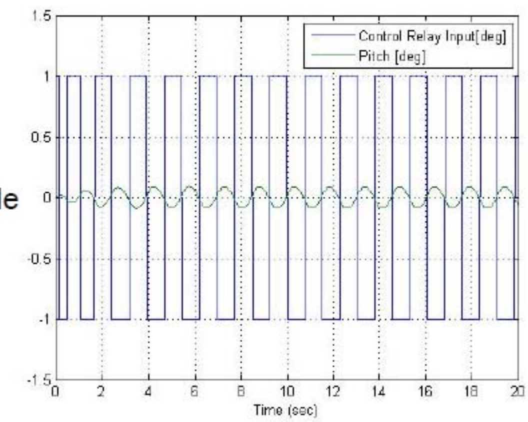
# Autotuning

1. Apply Relay feedback to find limit cycle




Measure *a* and $\omega$

*a* = Error max magnitude




$\omega$ = oscillation frequency

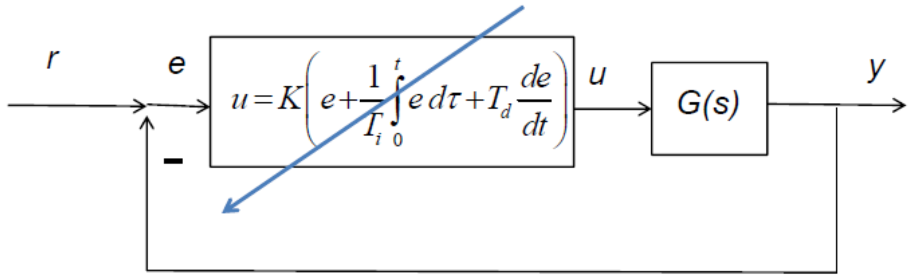
define

$$T = \frac{2\pi}{\omega}$$

$$K_a = \frac{4d}{\pi a}$$

## Autotuning

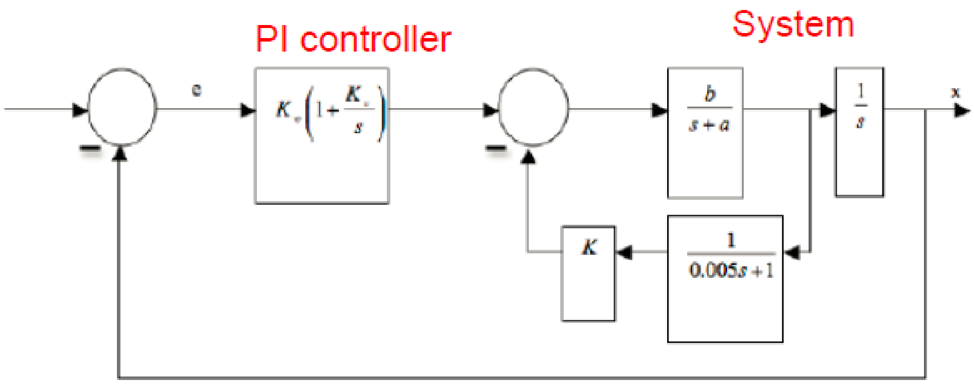
2. Compute PID gains using Zeigler-Nichols

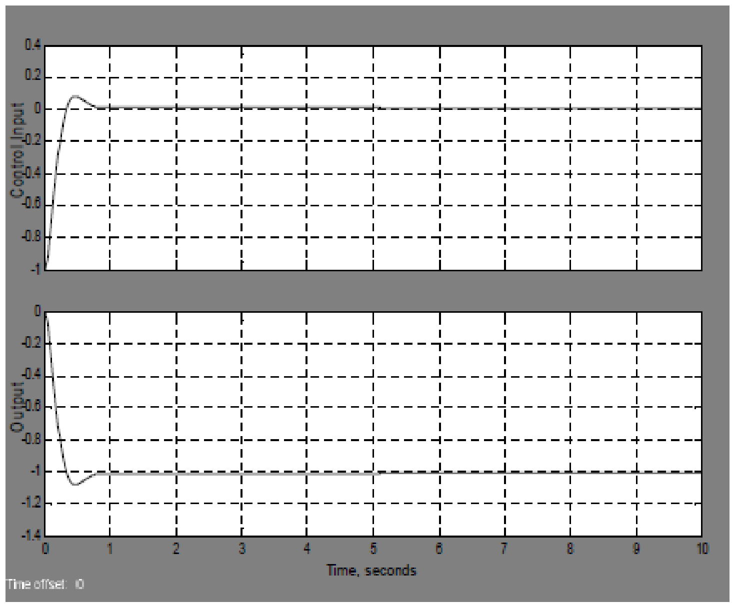



| | $K$ | $T_i$ | $T_d$ |
|---|---|---|---|
| P | $0.5K_a$ | | |
| PI | $0.4K_a$ | $0.8T$ | |
| PID | $0.6K_a$ | $0.5T$ | $0.12T$ |

3. Switch in PID controller and run process

Result using PI gains from Autotuning



## Implementing Analog Compensators

Analog compensators may be implemented using electric circuits as shown in the figure. However, today compensators are generally implemented using digital techniques on computers or microprocessors. We shall cover digital compensators later.

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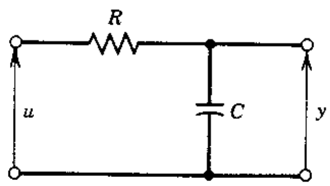
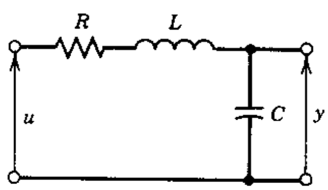
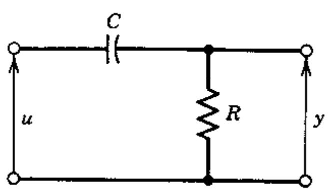
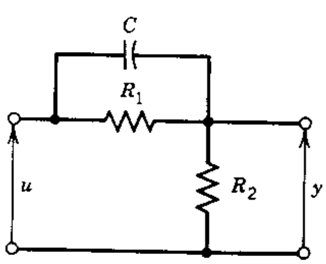
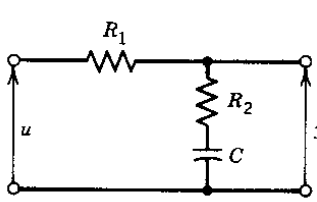
**TABLE 3.2-1. Network Transfer Functions and State Equations**

| Network | Transfer Function | State Equations |
|---|---|---|
|  Simple lag | $\frac{1}{1+s\tau}$, $\tau = CR$ | $\dot{x} = \frac{u-x}{\tau}$<br>$y = x$ |
|  Quadratic lag | $\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$<br>$\omega_n^2 = \frac{1}{LC}$<br>$\zeta = \frac{R}{2}\sqrt{\frac{C}{L}}$ | $\dot{x}_1 = x_2$<br>$\dot{x}_2 = -\omega_n^2 x_1 - 2\zeta\omega_n x_2 + \omega_n^2 u$<br>$y = x_1$ |
|  Simple lead | $\frac{s\tau}{1+s\tau}$, $\tau = CR$ | $\dot{x} = \frac{u-x}{\tau}$<br>$y = u - x$ |
|  Lead compensator | $\frac{s+z}{s+p}$, $z = 1/\tau$<br>$p = 1/(\alpha\tau)$<br>$\alpha = \frac{R_2}{R_1 + R_2}$<br>$\tau = CR_1$ | $\dot{x} = u - px$<br>$y = u + (z-p)x$ |
|  Lag compensator | $\alpha\left(\frac{s+z}{s+p}\right)$, $z = 1/(\alpha\tau)$<br>$p = 1/\tau$<br>$\alpha = \frac{R_2}{R_1 + R_2}$<br>$\tau = C(R_1 + R_2)$ | $\dot{x} = u - px$<br>$y = \alpha[u + (z-p)x]$ |