EE 4314 Homeworks and Exams
Spring 2012

- For full credit, show all work.
- Some problems require hand calculations. In those cases, do not use MATLAB except to check your answers.

EE 4314 Homework 1
Spring 2012

Simulation, MATLAB, System Response

1. Simulate the van der Pol oscillator $y''+\alpha(y^2-1)y'+y=0$ using MATLAB. Run for 100 sec. Plot y(t) vs. t and also the phase plane plot y'(t) vs. y(t). Use y(0)=0.1, y'(0)= 0.1.
   a. For $\alpha = 0.1$.
   b. For $\alpha = 0.8$.

2. Do MATLAB simulation of the Lorenz Attractor chaotic system. Run for 200 sec. with all initial states equal to 0.1. Plot states versus time, and also make 3-D plot of x_1, x_2, x_3 using PLOT3(x1,x2,x3).

$$\dot{x}_1 = -\sigma(x_1 - x_2)$$
$$\dot{x}_2 = rx_1 - x_2 - x_1 x_3$$
$$\dot{x}_3 = -bx_3 + x_1 x_2$$

use σ= 10, r= 28, b= 8/3.

3. An aircraft has a transfer function of $H(s) = \dfrac{1}{s^2 + 0.2s + 9.01}$.
   a. Plot poles and zeros.
   b. Find impulse response by hand.
   c. Use MATLAB to plot impulse response.
   d. Use MATLAB to plot step response

4. An automobile speed control transfer function is $H(s) = \dfrac{2.5(s+2)}{(s+5)(s+1)(s+3)}$
   e. Plot poles and zeros.
   f. Find impulse response by hand.
   g. Use MATLAB to plot impulse response.
   h. Use MATLAB to plot step response

State Variables and Block Diagrams

Show all work, do by hand except where stated.

1. An antenna pointing system can be written in SV form as

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ -6 & -7 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u = Ax + Bu, \quad y = \begin{bmatrix} 2 & 1 \end{bmatrix} x = Cx$$

   a. Find poles and natural modes.
   b. Find $\Phi(s)$ and $\phi(t)=e^{At}$.
   c. Find transfer function.
   d. Find impulse response.
   e. Find step response.
   f. Find output y(t) if $u(t)= 3e^{-2t}u_{-1}(t)$, $x(0)= [1 \quad -1]^T$.
   g. Plot step response using MATLAB.

2. A vehicle steering system has the block diagram
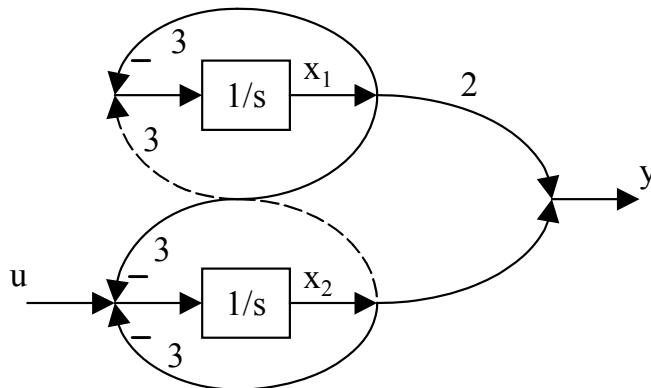
   a. Find SV representation (A,B,C,D).
   b. Find transfer function H(s) from (A,B,C,D).
   c. Plot impulse response using MATLAB.

3. A tank suspension system has dynamics $y''+4y'+5y = 2u'-u$. Find the output $y(t)$ if $u(t) = 2e^{-t}u_{-1}(t)$ and the ICs are $y(0) = 1$, $y'(0) = 1$.
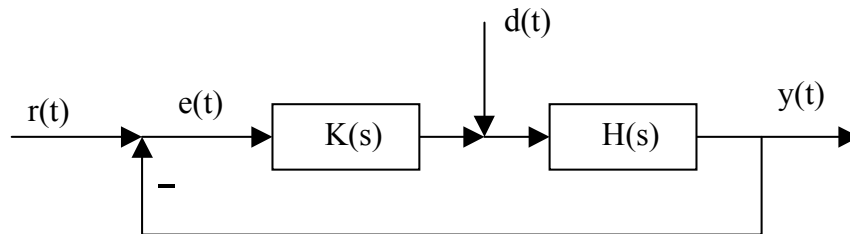
Feedback Design and Steady-State Error

1. A missile system during launch can be written in state-space form as

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

   a. Find open-loop poles.
   b. Find the state-variable feedback K in $u = -Kx$ so that the closed-loop system has damping ratio of $\varsigma = 1/\sqrt{2}$ and time constant of $\tau = 1\sec$.

2. An antenna pointing system is given by
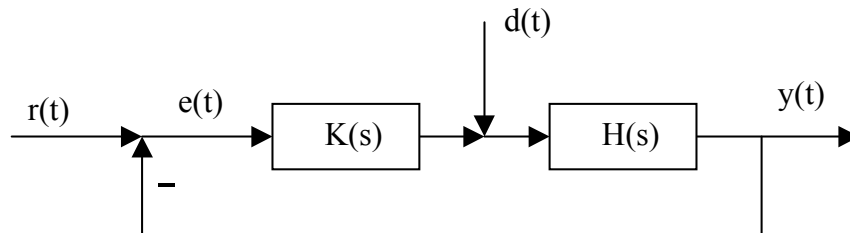

   with $r(t)$ the angle command and $d(t)$ the wind disturbance. The antenna has transfer function $H(s) = \dfrac{5}{s^2}$ and the compensator is the PD feedback $K(s) = k_d s + k_p$.

   a. Find transfer function from $r(t)$ to $y(t)$.
   b. Find closed-loop system characteristic polynomial
   c. Find PD gains k_d and k_p to make closed-loop system have $\omega_n = 5, \alpha = 4$.

3. An automobile speed control system is given by

with $r(t)$ the angle command and $d(t)$ the road disturbance, including road incline. The automobile has transfer function $H(s) = \dfrac{s+3}{s}$ and the compensator is $K(s) = k\dfrac{s+5}{s+2}$ with k a control gain.

    a. Find steady-state error to a unit step disturbance.

    b. Find steady-state error to a unit step command

    c. Find control gain k to make the steady-state error to a unit RAMP command $r(t)$ equal 5%. That is, we need steady-state error =0.05 in response to a velocity command.

LQR, Discretization of Controllers

1. Digital controller implementation
Let a continuous time controller be given by the transfer function

$$\frac{U(s)}{E(s)} = \frac{2}{(10s+1)} .$$

This continuous-time controller is difficult to implement on actual systems since it needs an integration routine to compute the control signal output from the error signal input.

a) Use the bilinear transformation to obtain the digital controller considering the sample time of T= 1sec.
b) Give a mathematical expression that will allow calculation of the control signal at time k based on measurements of past values of the error signal and past values of the control signal. That is, find a difference equation to compute the control.

2. **LQR Design.** An aircraft has a transfer function of. $H(s) = \dfrac{1}{s^2 + 0.2s + 9.01}$. This is too lightly damped. We want to design an LQR controller to improve the response.

 a. Plot open loop poles in the s-plane (by hand is OK.)
 Plot open-loop step response using MATLAB.
 b. Do an LQR design using $Q = I, R = 1$, with I the identity matrix.
 Plot closed-loop step response using MATLAB.
 Plot closed-loop poles.
 c. Do an LQR design using $Q = I, R = 0.01$
 Plot closed-loop step response using MATLAB.
 Plot closed-loop poles.

Root Locus

1. **Root Locus.** The plant is unstable and given by $H(s) = \dfrac{1}{(s-2)^2}$. We would like to obtain closed-loop stability. Use root locus to determine whether each of the following compensators K(s) can meet these specifications. If the compensator yields stability, find the range of gain for stability, and show simulation plots using MATLAB. Plot y(t) and y'(t) vs. time.

   a. Proportional control K(s)= k.

   b. PI control K(s)= k_p+k_I/s.

   c. PD control K(s)= k_ds+k_p.

   d. Realizable PD control K(s)= (k_ds+k_p)/(s+), with   large.

   e. PID control K(s)= k_ds+k_p+k_I/s.

   f. Lead compensator K(s)= k(s+a)/(s+b), a less than b.

   g. Lag compensator K(s)= k(s+a)/(s+b), a greater than b.

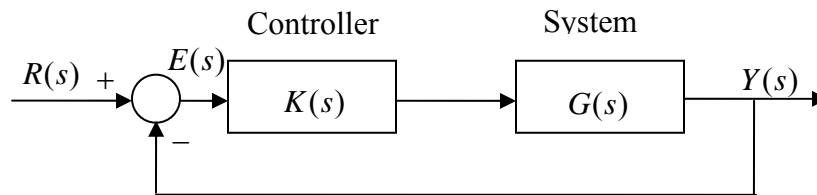2. **Root locus.** A closed loop system is presented in Figure 1.


Figure 1. Feedback control system

The controller is the PI feedback $K(s)=k_p\dfrac{\left(s+\dfrac{k_i}{k_p}\right)}{s}$ and the system transfer function is given as $G(s)=\dfrac{1}{(s^2+2s+10)}$.

a. Take $\dfrac{k_i}{k_p}=6$. Draw the root locus for the closed loop system as the value of the controller parameter k_p varies from 0 to infinity.

b. Calculate the range of k_p for which the closed loop system is stable.

3.   Bode Plot
A feedback control system is given by
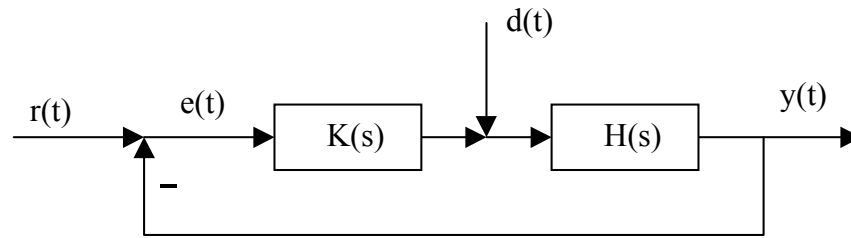
with *r(t)* an angle reference command and *d(t)* a disturbance. The system has transfer function
$H(s) = \dfrac{(s+5,000)}{(s+50)(s+500,000)}$ and the compensator is $K(s) = k\,\dfrac{s+5}{s(s+500)}$ with *k* a control gain.

Draw Bode plots of loop gain *H(s)K(s)*, magnitude and phase. (Hint, first find DC gain).
On the magnitude plot, indicate slopes. On the phase plot, indicate angles

4.   Bode Design
An aircraft longitudinal control system is given by
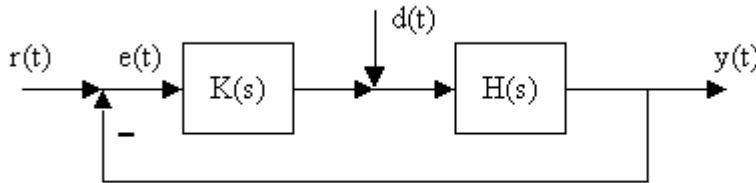
The plant is H(s)= $1/s(s^2+4s+8)$ and the compensator is the proportional gain K(s)= k.

  a.   Sketch by hand the Bode plots of the open-loop gain *KH(jω)*.
  b.   Plot Bode using MATLAB.
  c.   Sketch root locus. Use Routh to find range of gain for stability.
  d.   Sketch by hand the Bode plots of the closed-loop transfer function Y(s)/R(s).
  e.   Use MATLAB to make Bode plots of the closed-loop transfer function.