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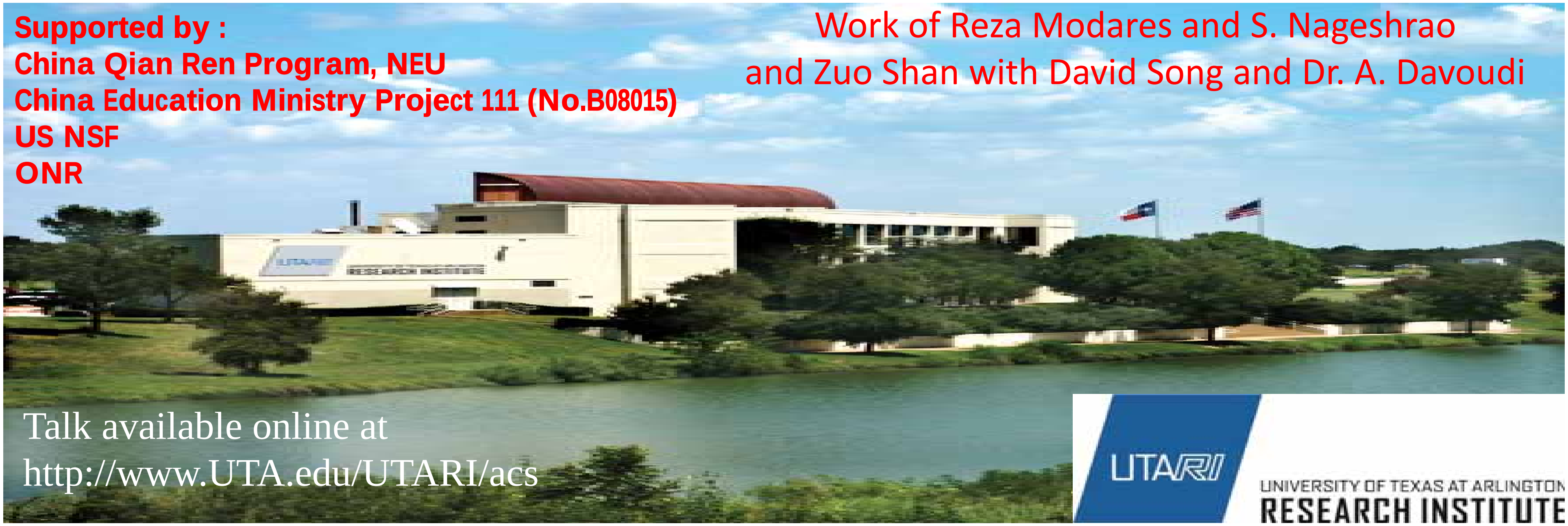
Output Regulation of Heterogeneous MAS- Reinforcement Learning Solution for Unknown Systems



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Work of Reza Modares and S. Nageshrao and Zuo Shan with David Song and Dr. A. Davoudi

Talk available online at
http://www.UTA.edu/UTARI/acs

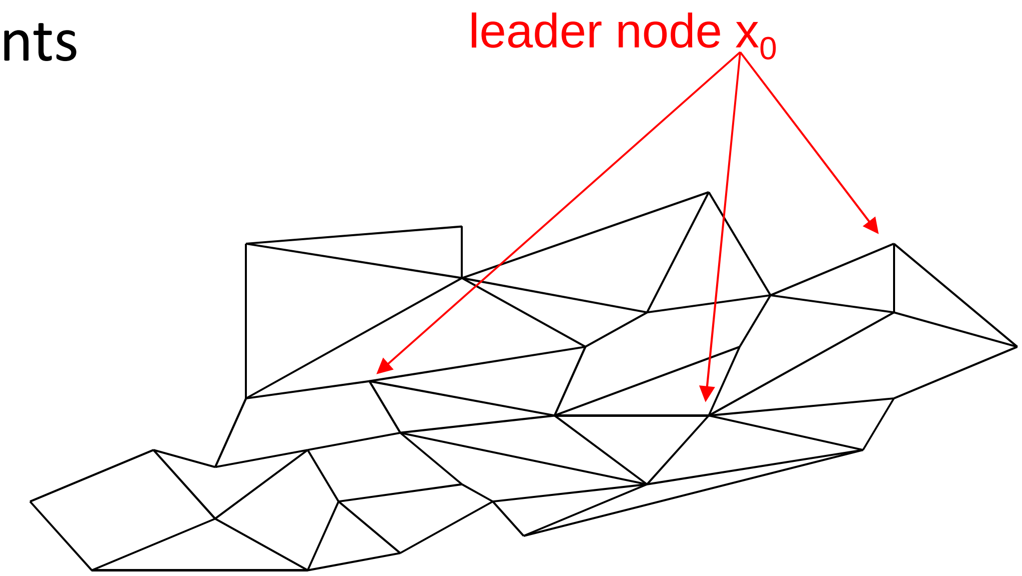
Output Synchronization of Heterogeneous MAS

Heterogeneous Multi-Agents

$$\dot{x}_i = A_i x_i + B_i u_i$$
$$y_i = C_i x_i$$

Leader

$$\dot{\zeta}_0 = S \zeta_0$$
$$y_0 = R \zeta_0$$



Output regulation error $\eta_i(t) = y_i(t) - y_0(t) \to 0$

Output regulator equations

$$A_i \Pi_i + B_i \Gamma_i = \Pi_i S$$
$$C_i \Pi_i = R$$

Dynamics are different, state dimensions can be different
o/p reg eqs capture the common core of all the agents dynamics
And define a synchronization manifold

Heterogeneous Multi-Agents

$$\dot{x}_i = A_i x_i + B_i u_i$$
$$y_i = C_i x_i$$

Leader

$$\dot{\zeta}_0 = S \zeta_0$$
$$y_0 = R \zeta_0$$

Output regulator equations
$$A_i \Pi_i + B_i \Gamma_i = \Pi_i S$$
$$C_i \Pi_i = R$$

Tracking error
$$\bar{\varepsilon}_i = \bar{x}_i - \Pi_i \zeta_0$$

Output regulation error
$$\eta_i(t) = y_i(t) - y_0(t) \to 0$$

Π_i is the insertion map of *S* in *A*

Dynamics are different, state dimensions can be different
o/p reg eqs capture the common core of all the agents dynamics
And define a synchronization manifold

Two Control Methods

Output regulator equations

$$A_i \Pi_i + B_i \Gamma_i = \Pi_i S$$

$$C_i \Pi_i = R$$

Control Method #1

$$\dot{\zeta}_i = S\zeta_i + c\left[\sum_{j=1}^{N} a_{ij}(\zeta_j - \zeta_i) + g_i(\zeta_0 - \zeta_i)\right]$$

$$u_i = K_{1i}(x_i - \Pi_i \zeta_i) + \Gamma_i \zeta_i = K_{1i} x_i + (\Gamma_i - K_{1i}\Pi_i)\zeta_i \equiv K_{1i} x_i + K_{2i}\zeta_i$$

Control Method #2- more intriguing

Local neighborhood output tracking error

$$e_{y_i} \equiv \sum_{j \in N_i} a_{ij}\left(y_j - y_i\right) + g_i\left(y_0 - y_i\right).$$

compensator

$$\begin{cases} \dot{z}_i = F_i z_i + G_i e_{y_i} \\ u_i = K_i x_i + H_i z_i \end{cases}$$

either

$$u_i = K_i x_i + H_i z_i = K_i x_i + (\Gamma_i - K_i \Pi_i) z_i$$

Or, assume p-copy in compensator
Then K_i, H_i are independent

Must know agent and leader's dynamics S,R

Data-driven Adaptive Solution of o/p Reg Eqs in Real Time

Output regulator equations

$$A_i \Pi_i + B_i \Gamma_i = \Pi_i S$$

$$C_i \Pi_i = R$$

Solve o/p regulator eqs online using measured data by Reinforcement Learning

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Brief paper

Optimal model-free output synchronization of heterogeneous systems using off-policy reinforcement learning☆

Hamidreza Modares[a], Subramanya P. Nageshrao[b,1], Gabriel A. Delgado Lopes[b], Robert Babuška[b], Frank L. Lewis[a,c]

Optimal Output Synchronization of Heterogeneous MAS Using Off-policy IRL

Nageshrao, Modares, Lopes, Babuska, Lewis

MAS $\dot{x}_i = A_i x_i + B_i u_i$

$y_i = C_i x_i$

Leader $\dot{\zeta}_0 = S \zeta_0$

$y_0 = R \zeta_0$

Optimal Tracker Problem

Augmented Systems

$$X(t) = \left[x_i(t)^T \;\; \zeta_0^{\;T} \right]^T \in \mathbb{R}^{n_i + p}$$

$$\dot{X}_i = T_i X_i + B_{1i} u_i$$

$$T_i = \begin{bmatrix} A_i & 0 \\ 0 & S \end{bmatrix}, B_{1i} = \begin{bmatrix} B_i \\ 0 \end{bmatrix}$$

Performance index

$$V(X_i(t)) = \int_t^{\infty} e^{-\gamma_i(\tau - t)} X_i^{\;T} (C_{1i}^{\;T} Q_i C_{1i} + K_i^{\;T} W_i K_i) X_i \, d\tau$$
$$= X_i(t)^T P_i X_i(t)$$

Control

$$u_i = K_{1i} x_i + K_{2i} \zeta_0 = K_i X_i$$

Optimal Tracker Solution by Reinforcement Learning

$$K_i = [K_{1i}, K_{2i}] = -W_i^{-1} B_{1i}^T P_i$$

Tracker ARE

$$T_i^T P_i + T_i P_i - \gamma_i P_i + C_{1i}^T Q_i C_{1i} - P_i B_{1i} W_i^{-1} B_{1i}^T P_i = 0$$

Algorithm 1. *On-policy IRL State-feedback algorithm*

Policy Evaluation- Solve IRL Bellman equation

$$e^{-\gamma_i \delta t} X_i(t+\delta t)^T P_i^\kappa X_i(t+\delta t) - X_i(t)^T P_i^\kappa X_i(t) = -\int_t^{t+\delta t} e^{-\gamma_i(\tau - t)} (y_i - y_0)^T Q_i (y_i - y_0) d\tau$$

Policy Update-

$$K_i^{\kappa+1} = [K_{1i}^{\kappa+1}, K_{2i}^{\kappa+1}] = -W_i^{-1} B_{1i}^T P_i^\kappa$$

Theorem- Algorithm 1 converges to the solution to the ARE

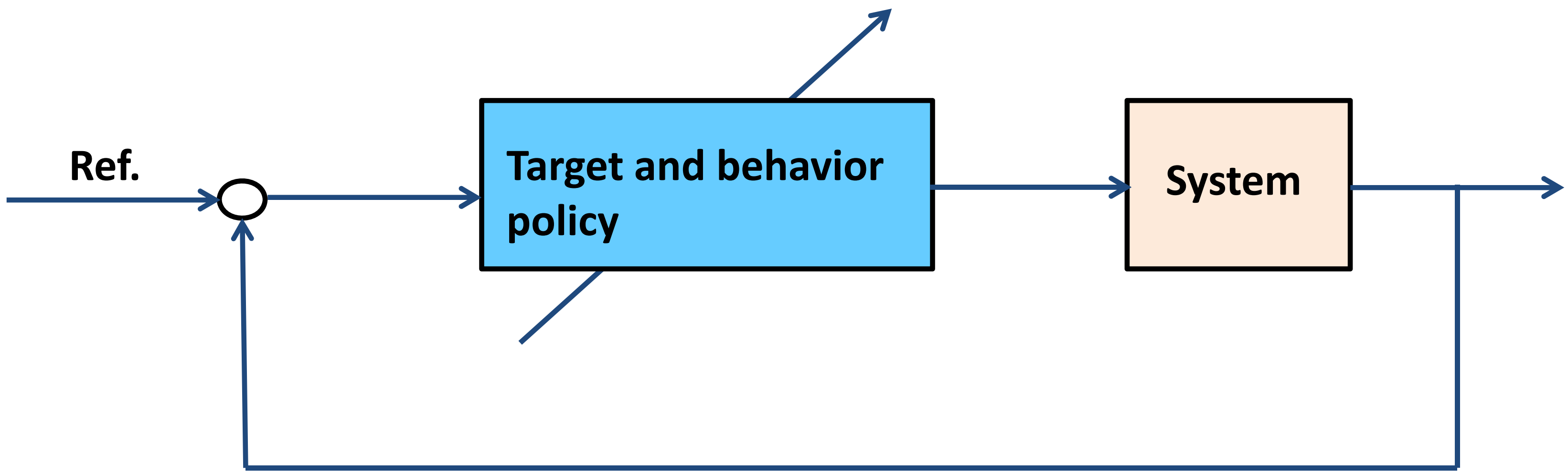
Bellman equation is solved using RLS or batch LS
It requires a Persistence of Excitation (PE) condition that may be hard to satisfy

Must know B_{1i}

On-policy RL

Target policy: The policy that we are learning about.

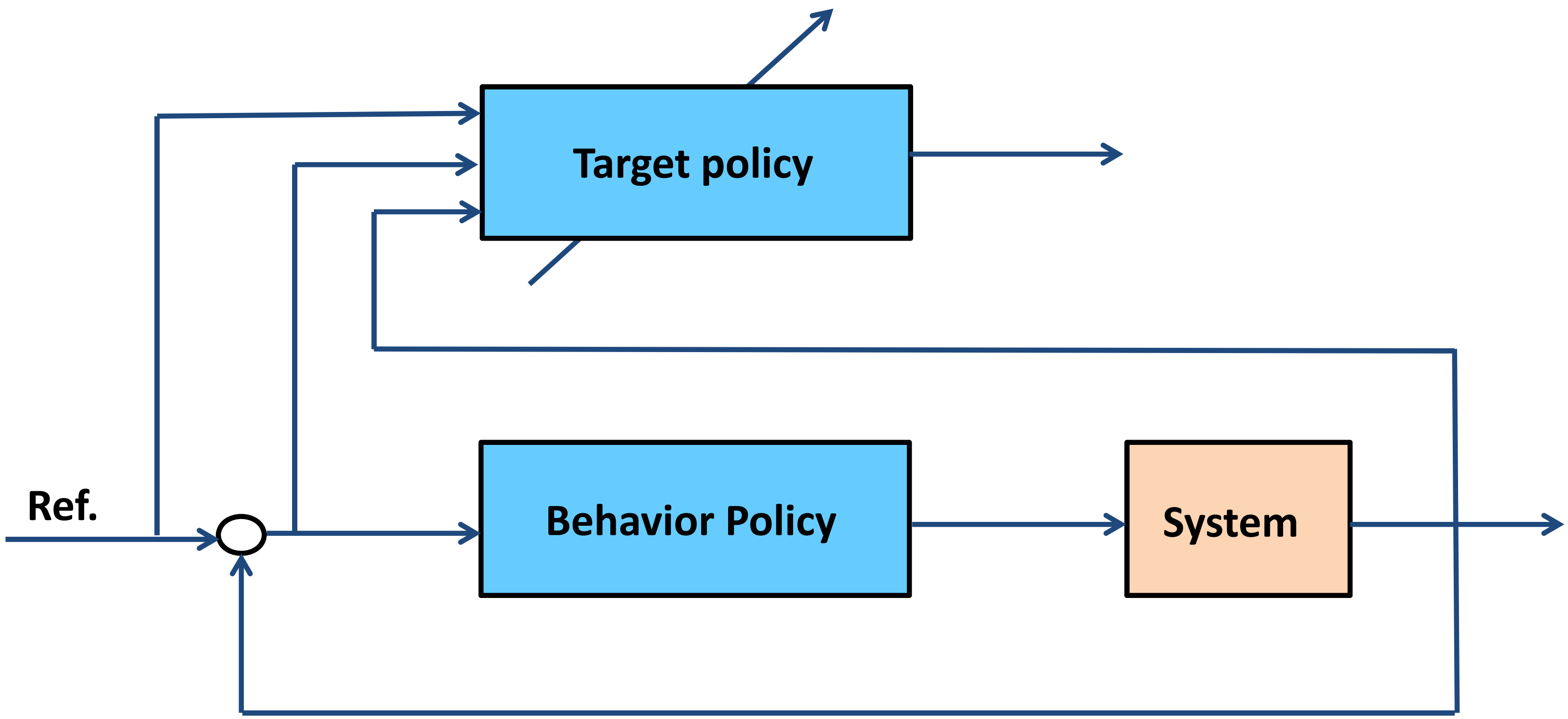
Behavior policy: The policy that generates actions and behavior



Target policy and behavior policy are the same

Off-policy RL

Humans can learn optimal policies while actually applying suboptimal policies



Target policy and behavior policy are different

Off-Policy RL

Tracker dynamics

$$\dot{X}_i = T_i X_i + B_{1i} u_i$$

Rewrite as

$$\dot{X}_i = (T_i + B_{1i} K_i^{\kappa}) X_i + B_{1i}(u_i - K_i^{\kappa} X_i) \equiv \bar{T}_i X_i + B_{1i}(u_i - K_i^{\kappa} X_i)$$

Now the Bellman equation becomes

$$e^{-\gamma_i \delta t} X_i(t+\delta t)^T P_i^{\kappa} X_i(t+\delta t) - X_i(t)^T P_i^{\kappa} X_i(t) = -\int_t^{t+\delta t} e^{-\gamma_i(\tau - t)} (y_i - y_0)^T Q_i (y_i - y_0) d\tau$$
$$+ \quad 2\int_t^{t+\delta t} e^{-\gamma_i(\tau - t)} (u_i - K_i^{\kappa} X_i)^T W_i K_i^{\kappa+1} X_i \, d\tau$$

Extra term containing $K_i^{\kappa+1}$

Algorithm 2. *Off-policy IRL Data-based algorithm*

Iterate on this equation and solve for $P_i^{\kappa}, K_i^{\kappa+1}$ simultaneously at each step

Note about probing noise If $u_i = K_i^{\kappa} X_i + e$ then $(u_i - K_i^{\kappa} X_i) = e$

Do not have to know any dynamics

agent $\dot{x}_i = A_i x_i + B_i u_i$, $y_i = C_i x_i$ Or leader $\dot{\zeta}_0 = S \zeta_0$, $y_0 = R \zeta_0$

Theorem- Off-policy Algorithm 2 converges to the solution to the ARE

$$T_i^T P_i + T_i P_i - \gamma_i P_i + C_{1i}^T Q_i C_{1i} - P_i B_{1i} W_i^{-1} B_{1i}^T P_i = 0$$

Theorem- o/p reg eq solution

Let $P_i = \begin{bmatrix} P^i_{11} & P^i_{12} \\ P^i_{21} & P^i_{22} \end{bmatrix}$

Then the solution to the output regulator equations

$$A_i \Pi_i + B_i \Gamma_i = \Pi_i S$$
$$C_i \Pi_i = R$$

Is given by

$$\Pi_i = -(P^i_{11})^{-1} P^i_{12}$$
$$\Gamma_i = K_{2i} - K_{1i} (P^i_{11})^{-1} P^i_{12}$$

Do not have to know the
Agent dynamics or the leader's dynamics (S,R)

Observer for Leader's State and Dynamics

To avoid knowledge of leader's state in

$$u_i = K_{1i} x_i + K_{2i} \zeta_0 = K_i X_i$$

Use adaptive observer for leader's state

$$\dot{\zeta}_i = \hat{S}_i \zeta_i + c \left[\sum_{j=1}^{N} a_{ij} (\zeta_j - \zeta_i) + g_i (\zeta_0 - \zeta_i) \right]$$

$$\dot{\hat{S}}_{\text{vec}i} = -\Gamma_{Si} (I_q \otimes \zeta_i) \left[\sum_{j=1}^{N} a_{ij} (\zeta_j - \zeta_i) + g_i (\zeta_0 - \zeta_i) \right]$$

Then use control

$$u_i = K_{1i} x_i + K_{2i} \zeta_i \equiv K_i \hat{X}_i \equiv K_i \begin{bmatrix} x_i \\ \zeta_i \end{bmatrix}$$

So this is the Control Method #1

Do not have to know the leader's dynamics (S,R)

Two Control Methods

Output regulator equations

$$A_i\Pi_i + B_i\Gamma_i = \Pi_i S$$

$$C_i\Pi_i = R$$

We just found a Controller #1 using data-based control

$$\dot{\zeta}_i = S\zeta_i + c\left[\sum_{j=1}^{N} a_{ij}(\zeta_j - \zeta_i) + g_i(\zeta_0 - \zeta_i)\right]$$

$$u_i = K_{1i}(x_i - \Pi_i\zeta_i) + \Gamma_i\zeta_i = K_{1i}x_i + (\Gamma_i - K_{1i}\Pi_i)\zeta_i \equiv K_{1i}x_i + K_{2i}\zeta_i$$

Now we seek a Controller using Method #2- more intriguing

Local neighborhood output tracking error

$$e_{y_i} \equiv \sum_{j\in N_i} a_{ij}\left(y_j - y_i\right) + g_i\left(y_0 - y_i\right).$$

compensator

$$\begin{cases} \dot{z}_i = F_i z_i + G_i e_{y_i} \\ u_i = K_i x_i + H_i z_i \end{cases}$$

either

$$u_i = K_i x_i + H_i z_i = K_i x_i + (\Gamma_i - K_i\Pi_i)z_i$$

Or, assume p-copy in compensator
Then K_i, H_i are independent

Do not want to solve o/p reg eqs or know any dynamics

Overall Dynamics Structure

Agents

$$\dot{x}_i = A_i x_i + B_i u_i + R_i x_0,$$
$$y_i = C_i x_i,$$

Leader

$$\dot{x}_0 = A_0 x_0,$$
$$y_0 = F x_0,$$

Compensator

$$\dot{z}_i = G_{1i} z_i + G_{2i} e_{iv},$$

Control Input

$$u_i = K_{y_i} y_i + K_{z_i} z_i$$

Local neighborhood output error

$$e_{iv} = \frac{1}{d_i + g_i}\left[\sum_{j \in N_i} e_{ij}(y_i - y_j) + g_i(y_i - y_0)\right],$$

Global form

$$e = \left[\left(diag\{\tfrac{1}{d_i+g_i}\}(L+G)\right) \otimes I_p\right] diag\{C_i\} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \end{bmatrix}$$

Global Dynamics

$$\varrho_g = [x_1^T, \ldots, x_N^T, z_1^T, \ldots, z_N^T]^T$$

$$r_G = [\underbrace{x_0^T, x_0^T, \ldots, x_0^T}_{N}]^T$$

$$\dot{\varrho}_g = A_{c_1}\varrho_g + B_c r_G,$$

$$\dot{r}_G = \tilde{A}_0 r_G,$$

$$e = [\tilde{C} \quad 0]\,\varrho_g - \tilde{F} r_G,$$

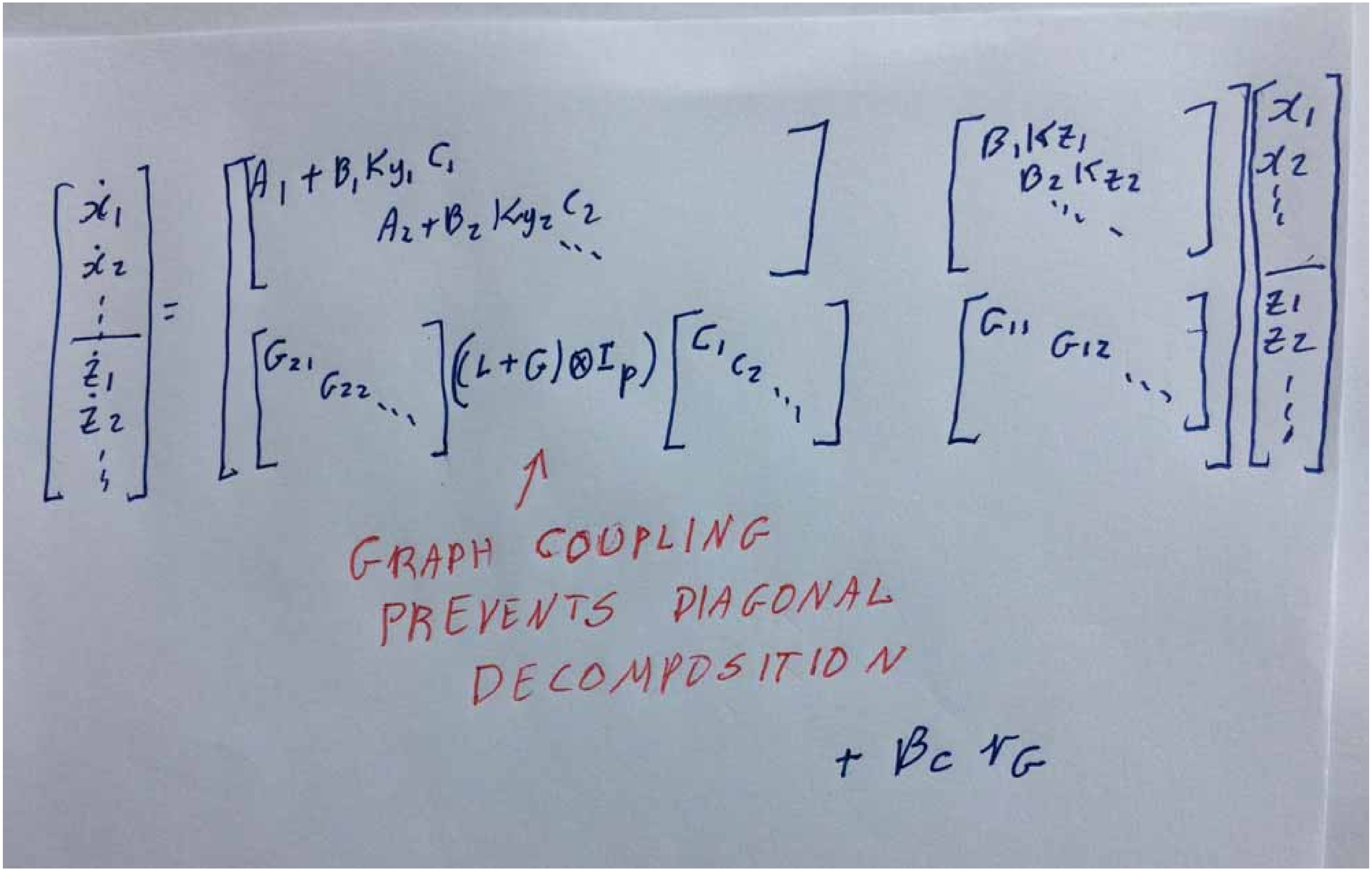
$$\tilde{A} = diag\{A_i\}$$

$$\begin{bmatrix} \dot{x} \\ \dot{z} \end{bmatrix}$$

$$A_{c_1} = \begin{bmatrix} \tilde{A} + \tilde{B}\tilde{K}_y\tilde{C} & \tilde{B}\tilde{K}_z \\ \tilde{G}_2\tilde{H}\tilde{C} & \tilde{G}_1 \end{bmatrix}, \qquad B_c = \begin{bmatrix} \tilde{R} \\ -\tilde{G}_2\tilde{H}\tilde{F} \end{bmatrix}.$$

$$\tilde{H} = \left(diag\{\tfrac{1}{d_i + g_i}\}(L+G) \right) \otimes I_p$$

Problems to Get Local Design Procedure




Joel Santos

A unified approach to output synchronization of heterogeneous multi-agent systems via $\mathcal{L}_2$-gain design ⋆

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Heterogeneous Multi-agent Dynamics

$$\begin{cases} \dot{x}_i = A_i x_i + B_i u_i, \\ y_i = C_i x_i, \end{cases}$$

Leader Dynamics

$$\begin{cases} \dot{\zeta}_0 = S\zeta_0, \\ y_0 = R\zeta_0, \end{cases}$$

Output Regulation Problem

$$\eta_i \equiv y_i - y_0 = C_i x_i - R\zeta_0.$$

$$\lim_{t \to \infty} \eta_i(t) = 0, \forall i \in N.$$

Local Neighborhood Output Error

$$e_{y_i} \equiv \sum_{j \in N_i} a_{ij} (y_j - y_i) + g_i (y_0 - y_i)$$

Compensator 1 – local state feedback

$$\begin{cases} \dot{z}_i = F_i z_i + G_i e_{y_i}, \\ u_i = K_i x_i + H_i z_i, \end{cases}$$

Compensator 2 – local output feedback

$$\begin{cases} \dot{z}_i = F_i z_i + G_i e_{y_i}, \\ u_i = K_i y_i + H_i z_i, \end{cases}$$

Compensator 3 – no local system feedback

$$\begin{cases} \dot{z}_i = F_i z_i + G_i e_{y_i}, \\ u_i = H_i z_i, \end{cases}$$

Local Systems with Interactions from Neighbors

Rewrite local o/p error

$$\begin{aligned} e_{y_i} &= \sum_{j\in N_i} a_{ij}\left(y_j - y_i\right) + g_i\left(y_0 - y_i\right) \\ &= -\left(d_i + g_i\right) y_i + \sum_{j\in N_i} a_{ij} y_j + g_i y_0 \\ &= -y_i + \sum_{j\in N_i} a_{ij} y_j + g_i y_0. \end{aligned}$$

Local Dynamics

$$\begin{bmatrix} \dot{x}_i \\ \dot{z}_i \end{bmatrix} = \begin{bmatrix} A_i + B_i K_i & B_i H_i \\ -G_i C_i & F_i \end{bmatrix} \begin{bmatrix} x_i \\ z_i \end{bmatrix}$$

Interaction terms

$$+\begin{bmatrix} 0 \\ G_i \end{bmatrix} \sum_{j\in N_i} a_{ij} \begin{bmatrix} C_j & 0 \end{bmatrix} \begin{bmatrix} x_j \\ z_j \end{bmatrix} + \begin{bmatrix} 0 \\ G_i \end{bmatrix} g_i R \zeta_0,$$

$$\eta_i = \begin{bmatrix} C_i & 0 \end{bmatrix} \begin{bmatrix} x_i \\ z_i \end{bmatrix} - R\zeta_0.$$

State Regulation Errors

$$\bar{\varepsilon}_i = \bar{x}_i - X_i \zeta_0$$

o/p reg eqs

$$\begin{cases} \bar{A}_i X_i + \bar{G}_i R = X_i S \\ \bar{C}_i X_i = R \end{cases}.$$

Error dynamics

$$\dot{\bar{\varepsilon}}_i = \bar{A}_i \bar{\varepsilon}_i + \bar{G}_i \sum_{j \in N_i} a_{ij} \bar{C}_j \bar{\varepsilon}_j$$

$$= \begin{bmatrix} A_i + B_i K_i & B_i H_i \\ -G_i C_i & F_i \end{bmatrix} \bar{\varepsilon}_i + \bar{G}_i \sum_{j \in N_i} a_{ij} \bar{C}_j \bar{\varepsilon}_j$$

$$= \begin{bmatrix} A_i + B_i K_i & B_i H_i \\ -G_i C_i & F_i \end{bmatrix} \bar{\varepsilon}_i + \bar{G}_i \bar{z}_i$$

$$\eta_i = \bar{C}_i \bar{\varepsilon}_i$$

Local transfer functions

$$T_i(s) \equiv \bar{C}_i \left(s - \bar{A}_i \right)^{-1} \bar{G}_i$$

State Regulation Errors

$$\bar{\varepsilon}_i = \bar{x}_i - X_i \zeta_0.$$

Error Dynamics

$$\begin{cases} \dot{\bar{\varepsilon}}_i = \bar{A}_i \bar{\varepsilon}_i + \bar{G}_i \sum_{j \in N_i} a_{ij} \bar{C}_j \bar{\varepsilon}_j, \\ \eta_i = \bar{C}_i \bar{\varepsilon}_i. \end{cases}$$

Global Form

$$\begin{cases} \dot{\zeta} = \operatorname{diag}(\bar{A}_i)\, \zeta + \operatorname{diag}(\bar{G}_i)\, \bar{\mathcal{A}} \operatorname{diag}(\bar{C}_i)\, \zeta, \\ \xi = \operatorname{diag}(\bar{C}_i)\, \zeta. \end{cases}$$

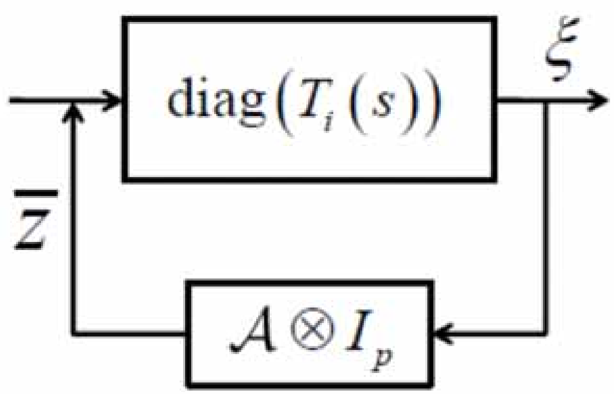


Figure 1. Closed-loop system of (10).

Main Theorem

Theorem 1. *For systems (16), suppose the following three conditions hold for each $i \in N$:*

(i) Matrix $\bar{A}_i$ is Hurwitz. Local Design

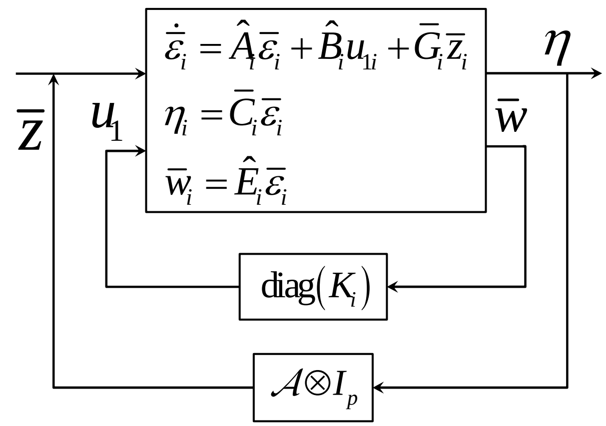
(ii) $\max_{i \in N} \|T_i\|_\infty < 1/\rho(\mathcal{A})$. Interaction Small Gain Condition

(iii) There exists a unique solution X_i to the output regulator equations

$$\begin{cases} \bar{A}_i X_i + \bar{G}_i R = X_i S, \\ \bar{C}_i X_i = R. \end{cases} \quad (17)$$

Then, Problem 1 is solved.

Local Design Procedure



$$\dot{\bar{\varepsilon}}_i = \bar{A}_i\bar{\varepsilon}_i + \bar{G}_i\sum_{j\in N_i} a_{ij}\bar{C}_j\bar{\varepsilon}_j$$

Closed-loop Systems

$$= \begin{bmatrix} A_i + B_iK_i & B_iH_i \\ -G_iC_i & F_i \end{bmatrix}\bar{\varepsilon}_i + \bar{G}_i\sum_{j\in N_i} a_{ij}\bar{C}_j\bar{\varepsilon}_j$$

$$= \begin{bmatrix} A_i + B_iK_i & B_iH_i \\ -G_iC_i & F_i \end{bmatrix}\bar{\varepsilon}_i + \bar{G}_i\bar{z}_i$$

Open-loop Systems

$$= \begin{bmatrix} A_i & 0 \\ -G_iC_i & F_i \end{bmatrix}\bar{\varepsilon}_i + \begin{bmatrix} B_i \\ 0 \end{bmatrix}\begin{bmatrix} K_i & H_i \end{bmatrix}\bar{\varepsilon}_i + \bar{G}_i\bar{z}_i$$

$$= \begin{bmatrix} A_i & 0 \\ -G_iC_i & F_i \end{bmatrix}\bar{\varepsilon}_i + \begin{bmatrix} B_i \\ 0 \end{bmatrix}\bar{K}_i\bar{\varepsilon}_i + \bar{G}_i\bar{z}_i.$$

Then problem is formulated as

$$\begin{cases} \dot{\bar{\varepsilon}}_i = \hat{A}_i\bar{\varepsilon}_i + \hat{B}_i u_{1i} + \bar{G}_i\bar{z}_i, \\ \eta_i = \bar{C}_i\bar{\varepsilon}_i \end{cases}$$

Static State feedback

$$u_{1i} = \bar{K}_i\bar{\varepsilon}_i$$

Optimal Local Design Algorithm

H-infinity design

Theorem 3. *Under Assumptions 1, 2, 3 and 4, select* $\gamma_i < 1/\rho(A)$. *Then, Problem 1 is solved if, for some positive definite matrices* R_i *and scalar* $\alpha_i > 0$, *one takes*

$$\bar{K}_i = -\frac{1}{\alpha_i} R_i^{-1} \hat{B}_i^T P_i$$

where $P_i^T = P_i > 0$ *is a solution of*

$$P_i \hat{A}_i + \hat{A}_i^T P_i + \bar{C}_i^T \bar{C}_i + \frac{1}{\gamma_i^2} P_i \bar{G}_i \bar{G}_i^T P_i - \frac{1}{\alpha_i} P_i \hat{B}_i R_i^{-1} \hat{B}_i^T P_i = 0.$$

[Gadewadikar et al., 2007] Gadewadikar, J., Lewis, F. L., Xie, L., Kucera, V., and Abu-Khalaf, M. (2007). Parameterization of all stabilizing H_∞ static state-feedback gains: application to output-feedback design. *Automatica*, 43(9):1597–1604.

Off-policy RL Algorithm to Solve Output Regulation for Heterogeneous MAS

$$
\begin{aligned}
& e^{-\delta_i \theta t}\bar{\varepsilon}_i(t+\theta t)^T P_i^\kappa \bar{\varepsilon}_i(t+\theta t) - \bar{\varepsilon}_i(t)^T P_i^\kappa \bar{\varepsilon}_i(t) \\
&= \int_t^{t+\theta t} e^{-\delta_i(\tau-t)}\left(-\eta_i^T Q_i \eta_i - \alpha_i \bar{\varepsilon}_i^T \left(\bar{K}_i^\kappa\right)^T R_i \bar{K}_i^\kappa \bar{\varepsilon}_i\right) d\tau \quad + \int_t^{t+\theta t} e^{-\delta_i(\tau-t)} \gamma_i^2 \left(\bar{z}_i^{\ \kappa}\right)^T \bar{z}_i^{\ \kappa} d\tau \\
&\quad - \int_t^{t+\theta t} e^{-\delta_i(\tau-t)} 2\alpha_i \bar{\varepsilon}_i^T \underline{\left(\bar{K}_i^{\kappa+1}\right)^T R_i \left(\bar{K}_i - \bar{K}_i^\kappa\right)} \bar{\varepsilon}_i d\tau \quad + \int_t^{t+\theta t} e^{-\delta_i(\tau-t)} 2\gamma_i^2 \left(\bar{z}_i^{\ \kappa+1}\right)^T \left(\bar{z}_i - \bar{z}_i^{\ \kappa}\right) d\tau.
\end{aligned}
$$

This is a standard parameter ID equation from Adaptive Control
It is linear in the unknown parameters
Solve using Batch LS or RLS to get the updated values

$$P_i^\kappa, \bar{K}_i^{\kappa+1}, \bar{z}_i^{\ \kappa+1}$$

This solves the o/p regulation problem for heterogeneous MAS
Without solving o/p reg equations and
without knowing any agent or leader's dynamics